Matrix Calculations

Assignment 7, Tuesday, March 21, 2017

Exercise teachers. Recall the following split-up of students:

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John van de Wetering	HG00.065	`wetering@cs.ru.nl`
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The delivery boxes are located in the Mercator 1 building on the ground floor (where the Computer Science department ICIS is located).

Handing in your answers: There are two options, *depending on your exercise class teacher*:

1. Delivery box (default): Put your solutions in the appropriate delivery box (see above). Before putting your solutions in the box make sure:
   - your name and student number are written clearly on the document.
2. E-mail (if your teacher agrees): Send your solutions by e-mail to your exercise class teacher (see above) with subject '*assignment 7*'. This e-mail should only contain a single PDF document as attachment (unless explicitly stated otherwise). Before sending an e-mail make sure:
   - the file is a PDF document that is well readable
   - your name is part of the filename (for example MyName_assignment-7.pdf)
   - your name and student number are included in the document (since they will be printed)

Deadline: Monday, March 27, 12:00 sharp!

Goals: After completing these exercises, you should be able to compute the inner product of vectors, cosine of the angle between vectors, check whether vectors are orthogonal, and turn a basis into an orthonormal basis using Gram-Schmidt orthogonalisation.

1. **(4 points)**

   Consider vectors:

   $$\boldsymbol{u} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \qquad \boldsymbol{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \qquad \boldsymbol{w} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

   compute the following quantities:

   (a) Inner products: $\langle \boldsymbol{u}, \boldsymbol{u} \rangle$, $\langle \boldsymbol{v}, \boldsymbol{v} \rangle$, $\langle \boldsymbol{w}, \boldsymbol{w} \rangle$, $\langle \boldsymbol{u}, \boldsymbol{v} \rangle$, and $\langle \boldsymbol{v}, \boldsymbol{w} \rangle$

   (b) Norms: $\|\boldsymbol{u}\|$, $\|\boldsymbol{v}\|$, $\|\boldsymbol{w}\|$

   (c) Distances: $d(\boldsymbol{u}, \boldsymbol{v})$, $d(\boldsymbol{v}, \boldsymbol{w})$

   (d) Normalised vectors: $\boldsymbol{u}' = a\boldsymbol{u}, \boldsymbol{v}' = b\boldsymbol{v}, \boldsymbol{w}' = c\boldsymbol{w}$ where $\|\boldsymbol{u}'\| = \|\boldsymbol{v}'\| = \|\boldsymbol{w}'\| = 1$

2. **(7 points)**

   Consider vectors:

   $$\boldsymbol{u} = \begin{pmatrix} 2 \\ 2 \\ 2+\lambda \end{pmatrix} \qquad \boldsymbol{v} = \begin{pmatrix} 1 \\ 1 \\ 1-\lambda \end{pmatrix}$$

(a) For which values of λ are $\boldsymbol{u}$ and $\boldsymbol{v}$ independent, and for which values are they orthogonal?

(b) Pick some λ such that these vectors are are independent, but **not** orthogonal. Use Gram-Schmidt orthgonalisation to make them orthogonal.

3. **(9 points)**

Consider the following basis for a subspace $U \subseteq \mathbb{R}^4$

$$\boldsymbol{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \qquad \boldsymbol{v}_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \qquad \boldsymbol{v}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}$$

(a) Use Gram-Schmidt orthogonalisation to turn this basis into an orthogonal basis $\mathcal{B}' = \{\boldsymbol{v}'_1, \boldsymbol{v}'_2, \boldsymbol{v}'_3\}$ for U.

(b) Normalise each of the vectors in $\mathcal{B}'$ to obtain an orthonormal basis for U.

(c) **BONUS:** Find a new vector $\boldsymbol{v}'_4$ such that $\mathcal{B}' = \{\boldsymbol{v}'_1, \boldsymbol{v}'_2, \boldsymbol{v}'_3, \boldsymbol{v}'_4\}$ is an orthogonal basis for all of $\mathbb{R}^4$. (Hint: $\boldsymbol{v}'_4$ is orthogonal to $\{\boldsymbol{v}'_1, \boldsymbol{v}'_2, \boldsymbol{v}'_3\}$ if and only if it is orthogonal to $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3\}$... Why?)