Matrix Calculations

Assignment 2, Wednesday, September 12, 2018

Exercise teachers. Recall the following split-up of students:

teacher	email	lecture room
Justin Reniers	`j.reniers@student.ru.nl`	E2.68 (E2.62 on 12 Oct)
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The delivery boxes are located in the Mercator 1 building on the ground floor (where the Computer Science department ICIS is located).

Handing in your answers: There are two options, *depending on your exercise class teacher*:

1. Delivery box (default): Put your solutions in the appropriate delivery box (see above). Before putting your solutions in the box make sure:
   - your name and student number are written clearly on the document.
2. E-mail (if your teacher agrees): Send your solutions by e-mail to your exercise class teacher (see above) with subject '*assignment 2*'. This e-mail should only contain a single PDF document as attachment (unless explicitly stated otherwise). Before sending an e-mail make sure:
   - the file is a PDF document that is well readable
   - your name is part of the filename (for example MyName_assignment-2.pdf)
   - your name and student number are included in the document (since they will be printed)

Deadline: Tuesday, September 18, 16:00 sharp!

Goals: After completing these exercises successfully you should be able to determine whether a set of vectors is linearly independent and whether a system of equations has zero, one, or many solutions. For homogeneous systems, you should be able to compute what its general solutions are. The total number of points is 20.

1. **(5 points)**

   Find the values of the parameter a and b such that the following system of linear equations:

   (i) has a unique solution,

   (ii) is inconsistent,

   (iii) has more than one solution.

$$\begin{aligned} x_1 + x_2 + ax_3 &= 2 \\ 2x_1 + x_2 + (2a+1)x_3 &= 5 \\ 3x_1 + (a-1)x_2 + 2x_3 &= b+2 \end{aligned}$$

   Hint: Perform Gaussian elimination where you keep parameter a and b in the matrix.

2. **(5 points)** Check if the following vectors are linearly dependent/independent. Explain your answers:

$$\text{(i)} \quad \left(\begin{pmatrix} 5 \\ 0 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 4 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 4 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 3 \\ 1 \end{pmatrix} \right) \qquad \text{(ii)} \quad \left(\begin{pmatrix} 1 \\ 5 \\ 4 \end{pmatrix}, \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} -14 \\ 20 \\ 7 \end{pmatrix} \right)$$

3. **(5 points)** A homogeneous system of linear equations is given:

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 &= 0 \\ 2x_1 + 4x_2 + 8x_3 + 10x_4 &= 0 \\ 3x_1 + 6x_2 + 11x_3 + 17x_4 &= 0 \end{aligned}$$

   (i) Perform Gauss elimination on the associated coefficient matrix to obtain an echelon form.

   (ii) Compute basic solution(s).

   (iii) Give the general solution in the format $(x_1, x_2, x_3, x_4) = c_1 \cdot \boldsymbol{v}_1 + \ldots + c_p \cdot \boldsymbol{v}_p$, where $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_p$ are the basic solution(s) that you've found under (ii).

4. **(0 points)** *Extra exercise, for those interested*

   Prove that the set of solutions of a homogeneous system of linear equations is closed under scalar multiplication.

   That is, show that if $(s_1, \ldots, s_n)$ is a solution of a homogeneous system of linear equations, then $c \cdot (s_1, \ldots, s_n)$ is also a solution (for any c).