Matrix Calculations

Assignment 4, Wednesday, September 26, 2018

Exercise teachers. Recall the following split-up of students:

teacher	email	lecture room
Justin Reniers	`j.reniers@student.ru.nl`	E2.68 (E2.62 on 12 Oct)
Justin Hende	`J.Hende@gmail.com`	HG00.062
Iris Delhez	`iam.delhez@student.ru.nl`	HG00.108
Stefan Boneschanscher	`S.Boneschanscher@student.ru.nl`	HG01.028
Serena Rietbergen	`serena.rietbergen@student.ru.nl`	HG02.028
Jen Dusseljee	`j.dusseljee@student.ru.nl`	HFML0220

The delivery boxes are located in the Mercator 1 building on the ground floor (where the Computer Science department ICIS is located).

Handing in your answers: There are two options, *depending on your exercise class teacher*:

1. Delivery box (default): Put your solutions in the appropriate delivery box (see above). Before putting your solutions in the box make sure:
   - your name and student number are written clearly on the document.
2. E-mail (if your teacher agrees): Send your solutions by e-mail to your exercise class teacher (see above) with subject '*assignment 4*'. This e-mail should only contain a single PDF document as attachment (unless explicitly stated otherwise). Before sending an e-mail make sure:
   - the file is a PDF document that is well readable
   - your name is part of the filename (for example MyName_assignment-4.pdf)
   - your name and student number are included in the document (since they will be printed)

Deadline: Tuesday, October 2, 16:00 sharp!

Goals: After completing this assignment, you should be able to determine if a set of vectors forms a basis, translate linear maps to/from matrices, and do matrix/vector multiplication.
The total number of points is 20.

1. **(6 points)** Determine if the following sets of vectors form a basis for the given vector space. If they do, prove that they are *linearly independent* and *spanning*. If they do not form a basis, explain why not.

   (i) $\left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\}$ for $\mathbb{R}^3$

   (ii) $\left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\}$ for $\mathbb{R}^3$

   (iii) $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 2 \\ 0 \end{pmatrix} \right\}$ for $V := \{(x, y, 2y, 0) \mid x, y \in \mathbb{R}\} \subseteq \mathbb{R}^4\}$

2. **(4 points)** Prove explicitly that the following maps are linear by checking that they preserve addition and scalar multiplication.

(i) $f\colon \mathbb{R}^3 \to \mathbb{R}^3$ defined by $f((x, y, z)) = (y + z, 2x + z, 3x - y + z)$.

(ii) $f\colon \mathbb{R}^3 \to \mathbb{R}^3$ defined by $f((x, y, z)) = (ax + by, cx + z, dx)$, for $a, b, c, d \in \mathbb{R}$.

3. **(2 points)** Show that the following maps are *not* linear

   (i) $f\colon \mathbb{R}^3 \to \mathbb{R}^2$ defined by $f((x, y, z)) = (x + z, y + xz)$;

   (ii) $f\colon \mathbb{R}^2 \to \mathbb{R}^2$ defined by $f((x, y)) = (x, y + 3)$.

4. **(4 points)** Consider the following matrices and vectors:

$$\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \qquad \boldsymbol{B} = \begin{pmatrix} 9 & 8 \\ 6 & 5 \\ 3 & 2 \end{pmatrix} \qquad \boldsymbol{v} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \qquad \boldsymbol{w} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

   Compute (i) $\boldsymbol{A} \cdot \boldsymbol{v}$, (ii) $\boldsymbol{B} \cdot \boldsymbol{w}$.

   Give intermediate results.

5. **(4 points)**

   This exercise is about transforming linear maps to/from matrices.

   (a) Give the matrix corresponding the linear map:

$$f((x_1, x_2, x_3, x_4)) = (x_1 + x_2 + 2x_4, 2x_1 + 3x_2 + x_3 + 6x_4, x_1 + x_4).$$

   in terms of the standard bases for $\mathbb{R}^4$ and $\mathbb{R}^3$.

   (b) Consider the following matrix, written in terms of the standard bases:

$$\boldsymbol{A} = \begin{pmatrix} 4 & 1 & 3 & -5 \\ 1 & 3 & 3 & 7 \end{pmatrix}$$

   give the linear map associated to $\boldsymbol{A}$.