

# Introduction to Tweakable Blockciphers

Bart Mennink

Radboud University (The Netherlands)

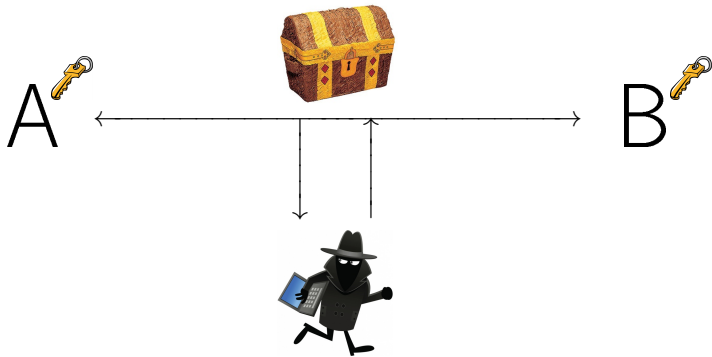
Summer school on real-world crypto and privacy

June 5, 2017

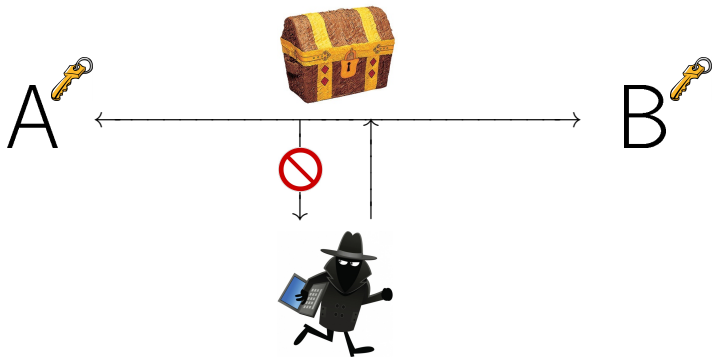
# Authenticated Encryption



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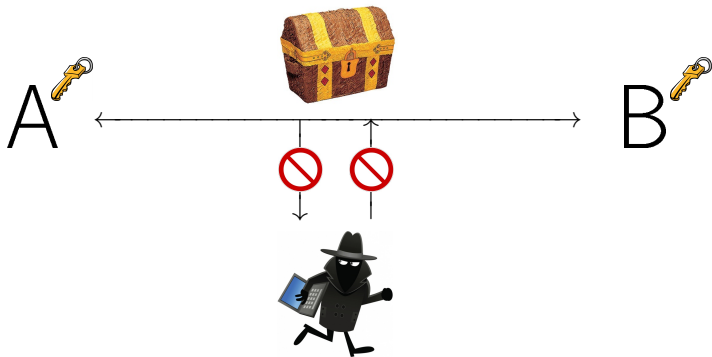
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## Encryption

- No outsider can learn anything about data

# Authenticated Encryption



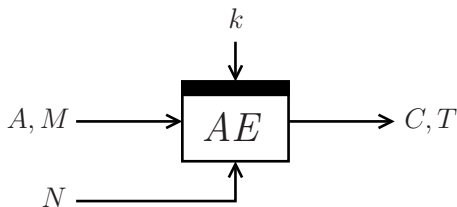
## Encryption

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## Authentication

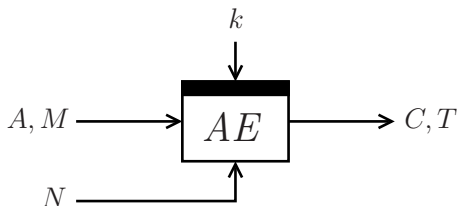
- No outsider can manipulate data

# Authenticated Encryption



- Ciphertext  $C$  encryption of message  $M$
- Tag  $T$  authenticates associated data  $A$  and message  $M$

# Authenticated Encryption



- Ciphertext  $C$  encryption of message  $M$
- Tag  $T$  authenticates associated data  $A$  and message  $M$
- Nonce  $N$  randomizes the scheme

# CAESAR Competition

## Competition for Authenticated Encryption: Security, Applicability, and Robustness

**Goal:** portfolio of authenticated encryption schemes

**Mar 15, 2014:** 57 first round candidates

**Jul 7, 2015:** 29.5 second round candidates

**Aug 15, 2016:** 16 third round candidates

**??:** announcement of finalists

**Dec 15, 2017:** announcement of final portfolio (?)





# CAESAR Competition, Not To Be Confused With:

ALL HAIL CAESAR.  
THE KING OF SALADS!  
ET TI, HOUSTON!

## CAESAR SALAD

## COMPETITION

THURSDAY, OCTOBER 6  
5:30 - 8 P.M.  
HILTON UNIVERSITY OF HOUSTON  
4450 UNIVERSITY DRIVE

TASTY? YES.  
GARLIC BREATH? INEVITABLE.  
FUN! ABSOLUTELY! FREE ADMISSION TO  
THE FIRST 10 GUESTS WHO WEAR A TOGA!

PURCHASE YOUR TICKETS  
\$40 IN ADVANCE • \$45 AT THE DOOR  
COMPLIMENTARY UNDERGROUND GARAGE PARKING  
[WWW.CAESARSALADCOMPETITIONHOUSTON.COM](http://WWW.CAESARSALADCOMPETITIONHOUSTON.COM)

PROCEEDS FROM THE EVENT BENEFIT THE FOOD & BEVERAGE  
MANAGERS ASSOCIATION EDUCATIONAL ENDOWMENTS.

"LETUCE\* DAZZLE YOU" WITH BOTH THE CLASSIC AND THE  
CREATIVE CULINARY ITERATIONS OF CAESAR SALADS AS CHEFS FROM THE  
HOUSTON AREA'S FINEST RESTAURANTS COMPETE FOR FOUR COWETED  
AWARDS—AND YOUR VOTE!

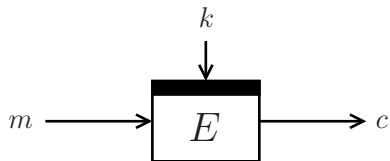
- CONSUMER'S CHOICE
- MOST CREATIVE
- BEST CLASSIC

UNIVERSITY of HOUSTON  
CONRAD N. HILTON COLLEGE

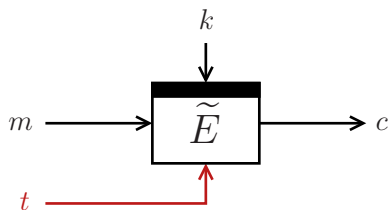
HOUSTON'S DINING MAGAZINE  
**MY TABLE**

FOOD & BEVERAGE  
FR FR  
Sponsored by: Bill Givner

# Tweakable Blockciphers

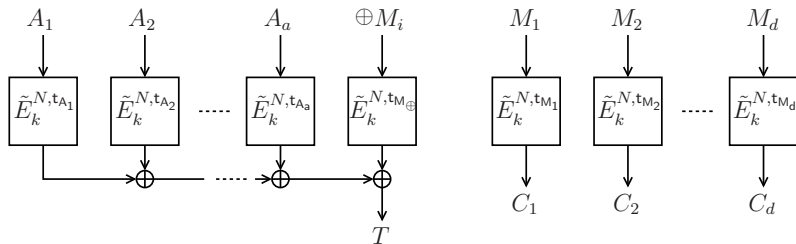


# Tweakable Blockciphers



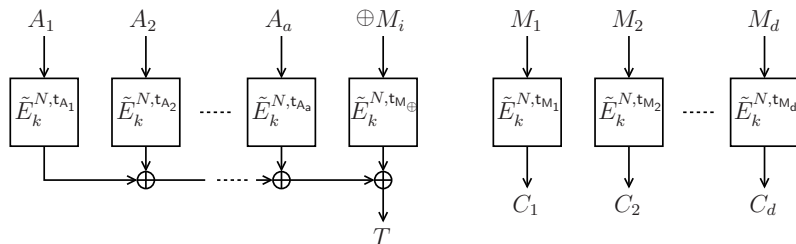
- Tweak: flexibility to the cipher
- Each tweak gives different permutation

## Tweakable Blockciphers in OCBx



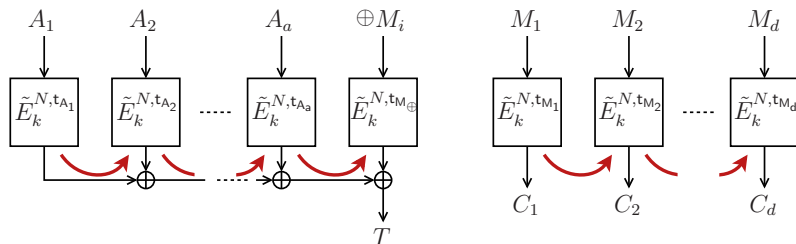
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# Tweakable Blockciphers in OCBx



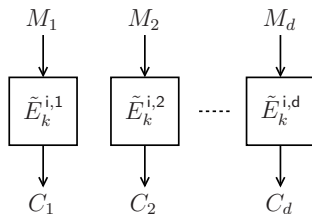
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- Internally based on tweakable blockcipher  $\tilde{E}$ 
  - Tweak  $(N, \text{tweak})$  is unique for **every** evaluation
  - Different blocks always transformed under different tweak

# Tweakable Blockciphers in OCBx



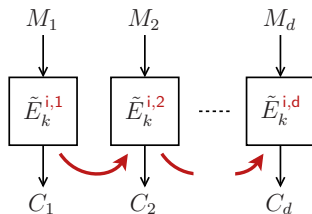
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## Tweakable Blockciphers in XTS



- XTS mode for disk encryption
- Tweak  $(i,j) = (\text{sector}, \text{block})$  unique for **every** block

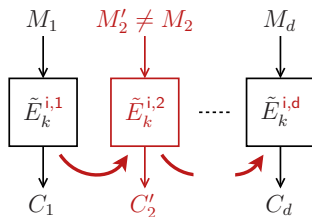
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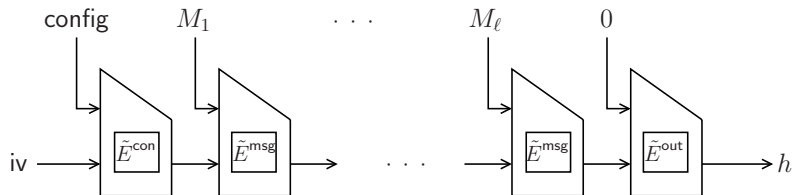


# Tweakable Blockciphers in XTS



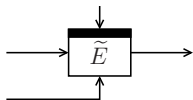
- XTS mode for disk encryption
- Tweak  $(i,j) = (\text{sector}, \text{block})$  unique for **every** block
- Change of tweak should be **efficient** (as before)
- **Incrementality**: change in one (or few) blocks

## Tweakable Blockciphers in Skein

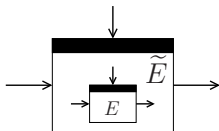


- Skein hash function by Ferguson et al. [FLS+07]
- Based on Threefish tweakable blockcipher
- Tweaks used for domain separation

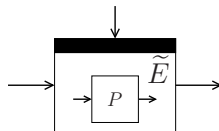
# Tweakable Blockcipher Designs



**Dedicated**

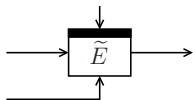


**Blockcipher-Based**



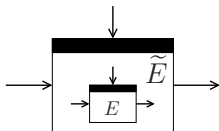
**Permutation-Based**

# Tweakable Blockcipher Designs in CAESAR



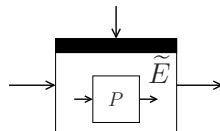
## Dedicated

KIASU,  
Joltik,  
SCREAM,  
Deoxys



## Blockcipher-Based

CBA, COBRA, iFeed,  
Marble, OMD, POET,  
SHELL, AEZ, COPA/  
ELmD, OCB, OTR



## Permutation-Based

Prøst,  
Minalpher

# Outline

Dedicated Design

Basic Generic Recipe

Tweakable Blockciphers Based on Masking

Beyond Masking-Based Tweakable Blockciphers

Conclusion

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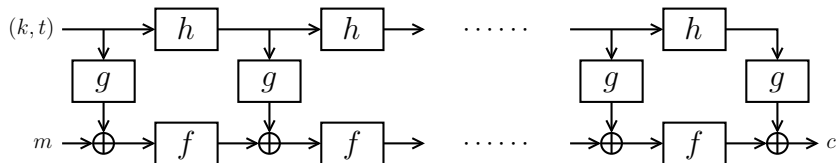
Conclusion

# Dedicated Tweakable Blockciphers

- Hasty Pudding Cipher [Sch98]
  - AES submission, “first tweakable cipher”
- Mercy [Cro01]
  - Disk encryption
- Threefish [FLS+07]
  - SHA-3 submission Skein
- TWEAKEY framework [JNP14]
  - Four CAESAR submissions
  - SKINNY & MANTIS

# TWEAKEY Framework

- TWEAKEY by Jean et al. [JNP14]:

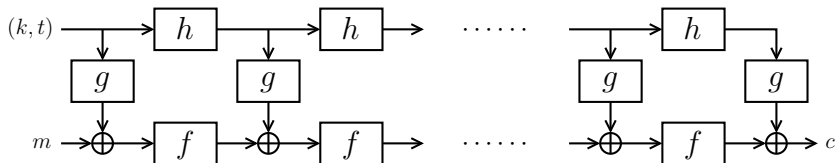


- $f$ : round function
- $g$ : subkey computation
- $h$ : transformation of  $(k, t)$



# TWEAKEY Framework

- TWEAKEY by Jean et al. [JNP14]:



- $f$ : round function
- $g$ : subkey computation
- $h$ : transformation of  $(k, t)$
- Security measured through cryptanalysis
- Our focus: modular design

# Outline

Dedicated Design

Basic Generic Recipe

Tweakable Blockciphers Based on Masking

Beyond Masking-Based Tweakable Blockciphers

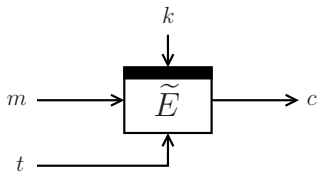
Conclusion

## Basic Generic Recipe



- 1 Determine appropriate security model
- 2 Design the scheme
- 3 Perform security analysis

## Basic Generic Recipe Step 1: Security Model



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### Tweakable Pseudorandom Permutation Security

- $\tilde{E}_k$  should look like random permutation for every  $t$
- Different tweaks  $\rightarrow$  pseudo-independent permutations

## Basic Generic Recipe Step 1: Security Model



### Tweakable Pseudorandom Permutation Security

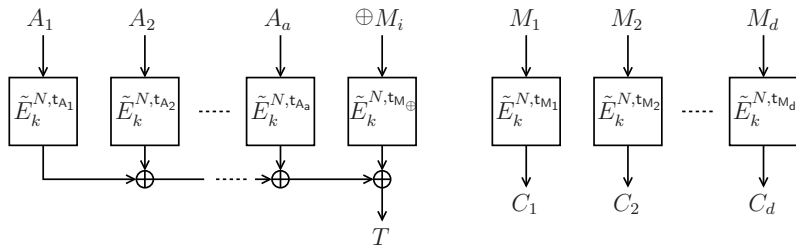
- $\tilde{E}_k$  should look like random permutation for every  $t$
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### Strong Tweakable Pseudorandom Permutation Security

- Adversary may have encryption and decryption access to  $\tilde{E}$

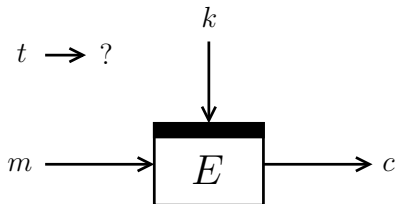
# Basic Generic Recipe Step 1: Security Model

## Example



- Tag generation:  $\tilde{E}_k$  evaluated in forward direction only
- Encryption/decryption:  $\tilde{E}_k$  evaluated in both directions

## Basic Generic Recipe Step 2: Playground

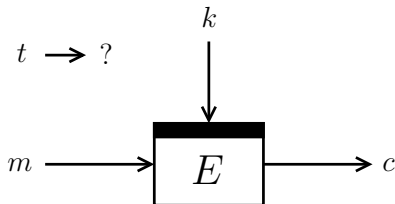


- Consider a blockcipher  $E$  with  $\kappa$ -bit key and  $n$ -bit state

How to mingle the tweak into the evaluation?



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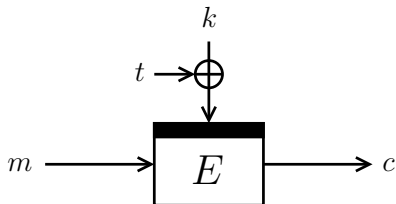
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blend it with the key

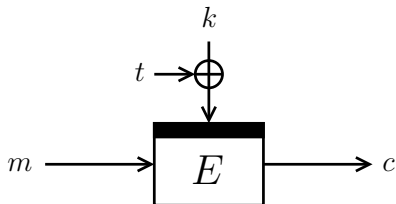
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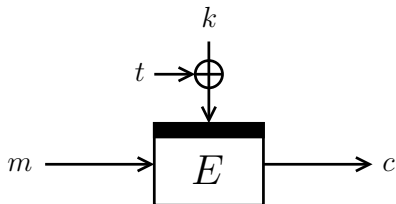
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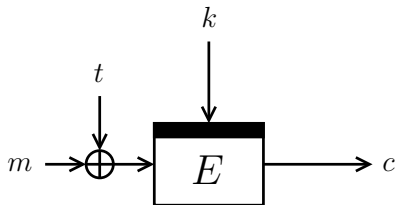
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- For  $\oplus$ -mixing, key can be recovered in  $2^{\kappa/2}$  evaluations
- Scheme is insecure if  $E$  is Even-Mansour

## Basic Generic Recipe Step 2: Playground



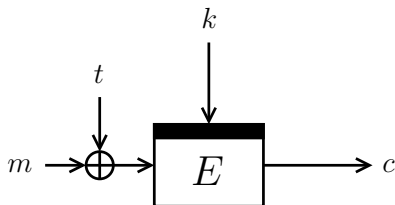
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- TWEAKEY blending is **more advanced**

## Basic Generic Recipe Step 2: Playground



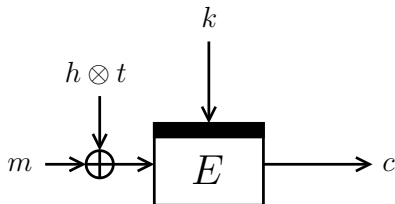
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## Basic Generic Recipe Step 2: Playground



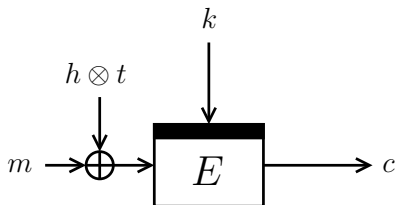
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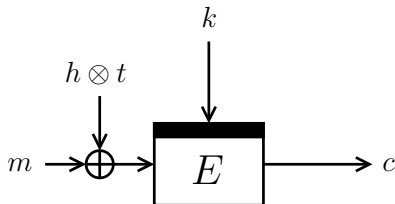
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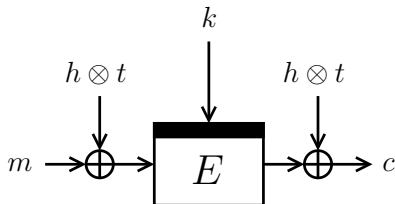


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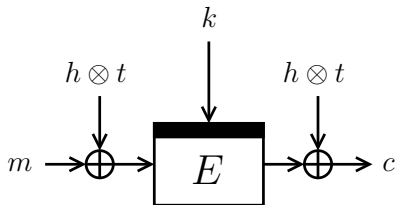
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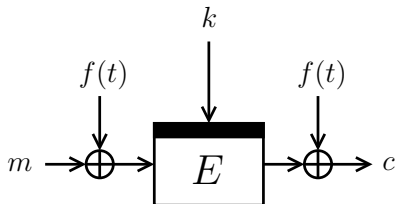
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  - Two-sided masking necessary

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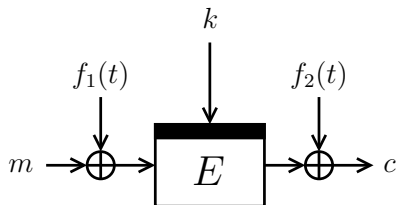
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- Can we generalize?

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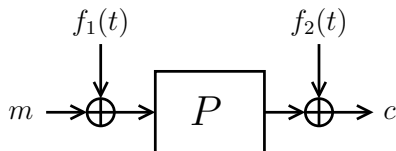
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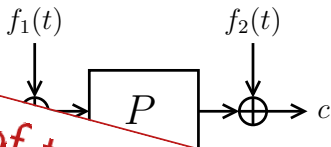
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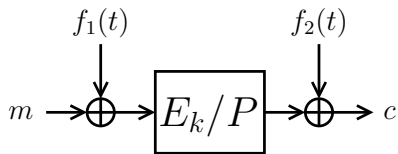
## Basic Generic Recipe Step 2: Playground



Majority of tweakable blockciphers follow mask- $E_k/P$ -mask principle

- Two-sided secret key
- Can we generalize?
- Generalizing masking? Depends on functions  $f_1, f_2$
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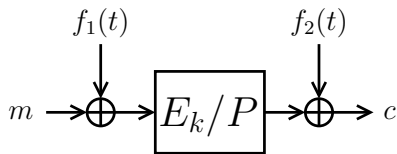
## Basic Generic Recipe Step 3: Analysis



- $\tilde{E}_k$  should “look like” random permutation for every  $t$
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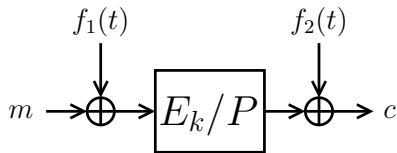


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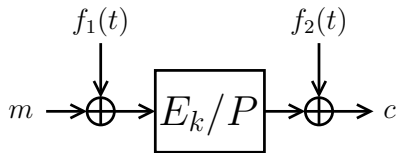
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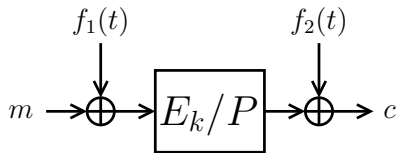


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- Step 3b:
  - How many evaluations does  $\mathcal{A}$  need **at least**?
  - Boils down to provable security

## Basic Generic Recipe Step 3a: Generic Attack



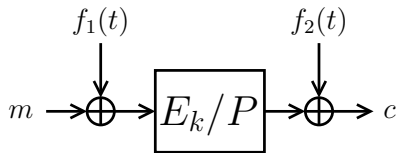
## Basic Generic Recipe Step 3a: Generic Attack



- For any two queries  $(t, m, c)$ ,  $(t', m', c')$ :

$$m \oplus f_1(t) = m' \oplus f_1(t') \implies c \oplus f_2(t) = c' \oplus f_2(t')$$

## Basic Generic Recipe Step 3a: Generic Attack

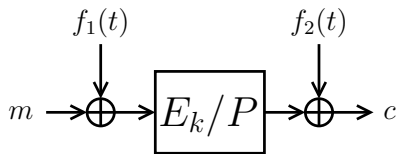


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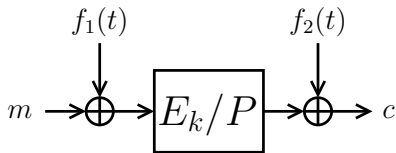


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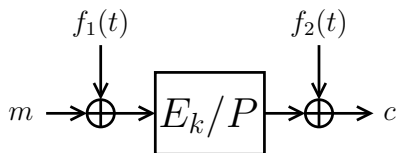
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- Unlikely to happen for random family of permutations
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Scheme can be broken in  $\approx 2^{n/2}$  evaluations

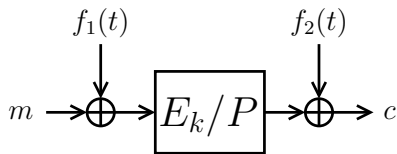
## Basic Generic Recipe Step 3b: Security Proof



- The fun starts here!
- More technical and often more involved

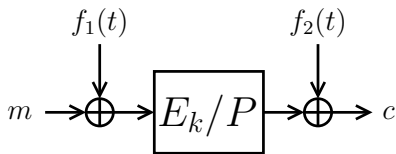


## Basic Generic Recipe Step 3b: Security Proof



- The fun starts here!
- More technical and often more involved
- Typical approach:
  - Consider any transcript  $\tau$  an adversary may see
  - Most  $\tau$ 's should be equally likely in both worlds
  - Odd ones should happen with very small probability

## Basic Generic Recipe Step 3b: Security Proof



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All constructions in this presentation: secure up to  $\approx 2^{n/2}$  evaluations

# Outline

Dedicated Design

Basic Generic Recipe

Tweakable Blockciphers Based on Masking

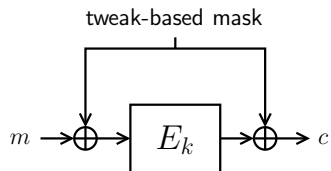
- State of the Art
- Improved Efficiency
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Beyond Masking-Based Tweakable Blockciphers

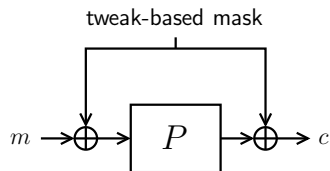
Conclusion

# Tweakable Blockciphers Based on Masking

## Blockcipher-Based

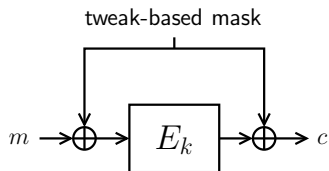


## Permutation-Based



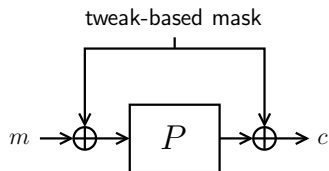
# Tweakable Blockciphers Based on Masking

## Blockcipher-Based



typically 128 bits

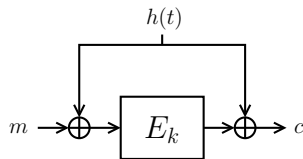
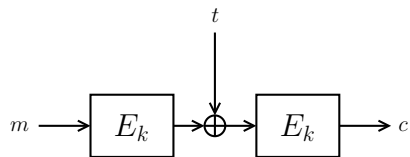
## Permutation-Based



much larger: 256-1600 bits

# Original Constructions

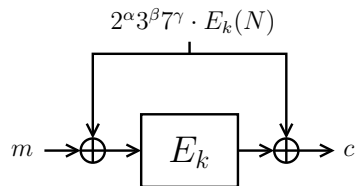
- $LRW_1$  and  $LRW_2$  by Liskov et al. [LRW02]:



- $h$  is XOR-universal hash
  - E.g.,  $h(t) = h \otimes t$  for  $n$ -bit “key”  $h$

## Powering-Up Masking (XEX)

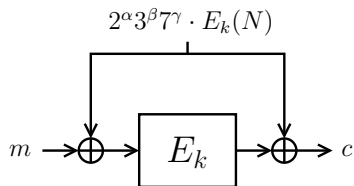
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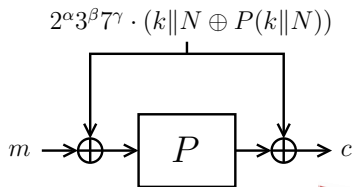
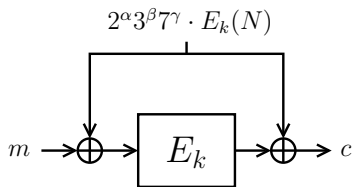


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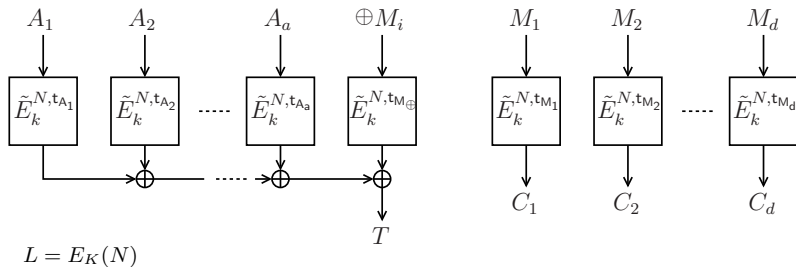
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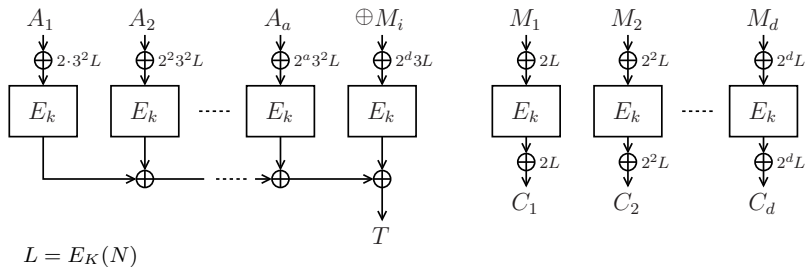


- $(\alpha, \beta, \gamma, N)$  is tweak (simplified)
- Used in OCB2,  $\pm 14$  CAESAR candidates, and XTS
- Permutation-based variants in Minalpher and Prøst (generalized by Cogliati et al. [CLS15])

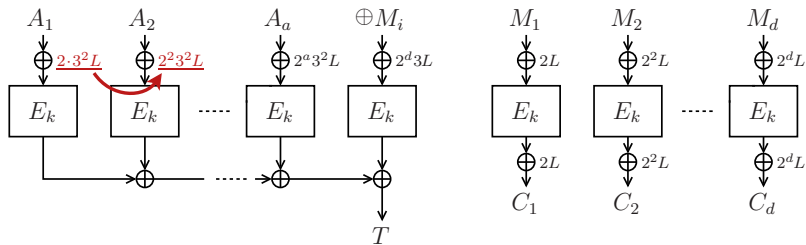
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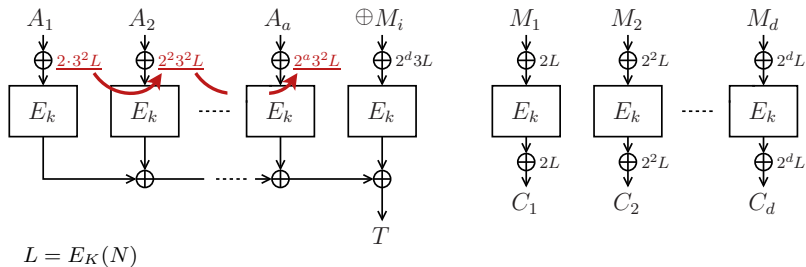


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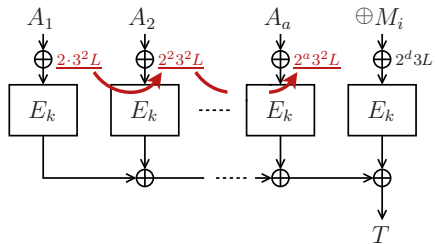


$$L = E_K(N)$$

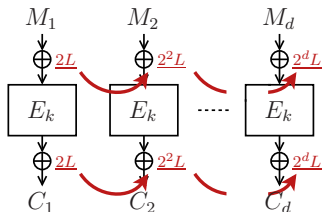
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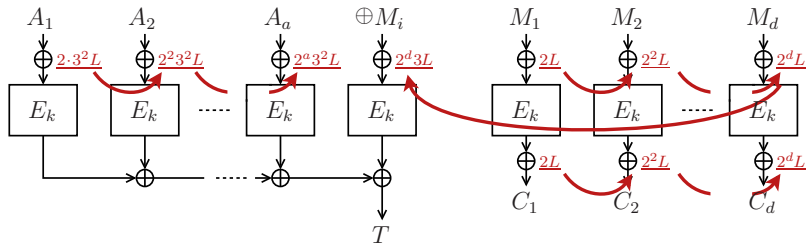
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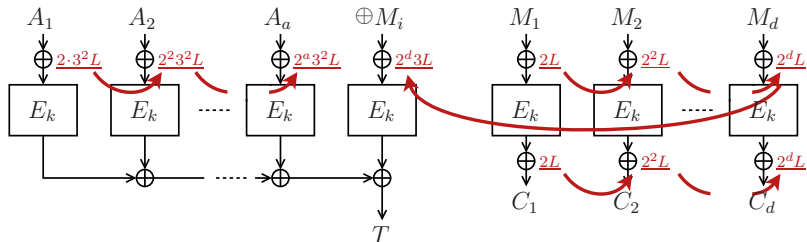
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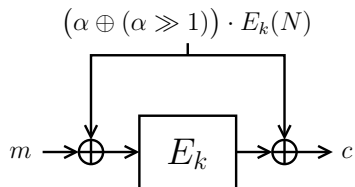
$$L = E_K(N)$$

- Update of mask:
  - Shift and conditional XOR
- Variable time computation
- Expensive on certain platforms



# Gray Code Masking

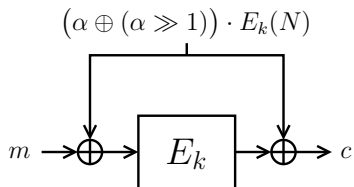
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- OCB1 and OCB3 use Gray Codes:



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- Updating:  $G(\alpha) = G(\alpha - 1) \oplus 2^{\text{ntz}(\alpha)}$ 
  - Single XOR
  - Logarithmic amount of field doublings (precomputed)
- More efficient than powering-up [KR11]

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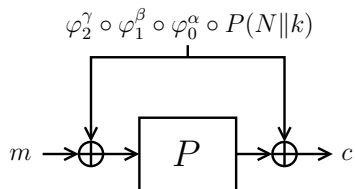
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Beyond Masking-Based Tweakable Blockciphers

Conclusion

# Masked Even-Mansour (MEM)

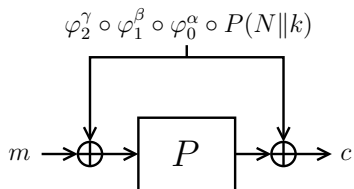
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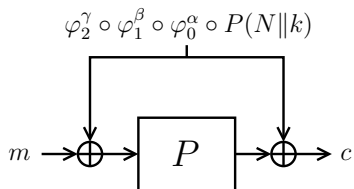
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- Combines advantages of:
  - Powering-up masking
  - Word-based LFSRs
- Simpler, constant-time (by default), more efficient

## MEM: Design Considerations

- Particularly suited for large states (permutations)
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$b$	$w$	$n$	$\varphi$
128	8	16	$(x_1, \dots, x_{15}, (x_0 \lll 1) \oplus (x_9 \ggg 1) \oplus (x_{10} \ll 1))$
128	32	4	$(x_1, \dots, x_3, (x_0 \lll 5) \oplus x_1 \oplus (x_1 \ll 13))$
128	64	2	$(x_1, (x_0 \lll 11) \oplus x_1 \oplus (x_1 \ll 13))$
256	64	4	$(x_1, \dots, x_3, (x_0 \lll 3) \oplus (x_3 \ggg 5))$
512	32	16	$(x_1, \dots, x_{15}, (x_0 \lll 5) \oplus (x_3 \ggg 7))$
512	64	8	$(x_1, \dots, x_7, (x_0 \lll 29) \oplus (x_1 \ll 9))$
1024	64	16	$(x_1, \dots, x_{15}, (x_0 \lll 53) \oplus (x_5 \ll 13))$
1600	32	50	$(x_1, \dots, x_{49}, (x_0 \lll 3) \oplus (x_{23} \ggg 3))$
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- Work exceptionally well for ARX primitives

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- Intuitively, masking goes well as long as

$$\varphi_2^\gamma \circ \varphi_1^\beta \circ \varphi_0^\alpha \neq \varphi_2^{\gamma'} \circ \varphi_1^{\beta'} \circ \varphi_0^{\alpha'}$$

for any  $(\alpha, \beta, \gamma) \neq (\alpha', \beta', \gamma')$

- Challenge: set proper domain for  $(\alpha, \beta, \gamma)$
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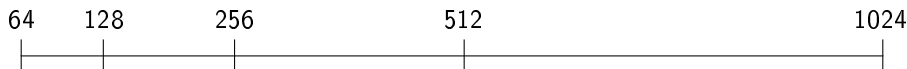
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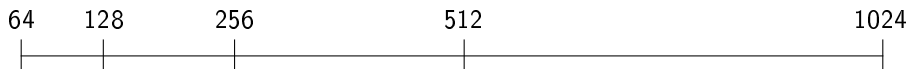
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solved by

Rogaway [Rog04]

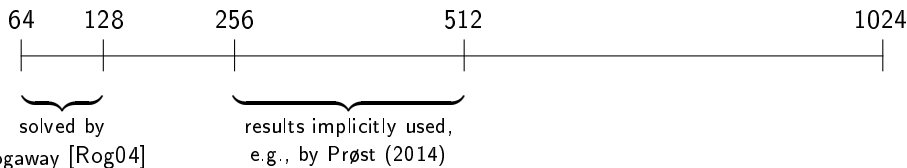
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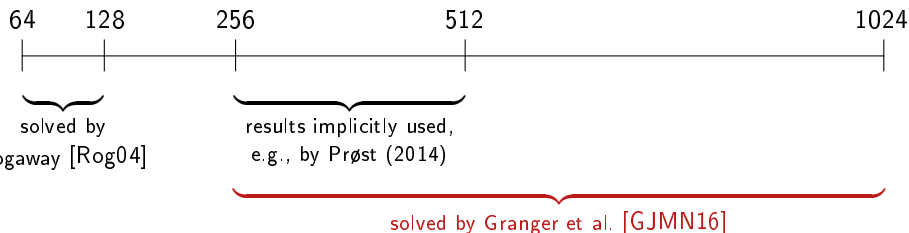
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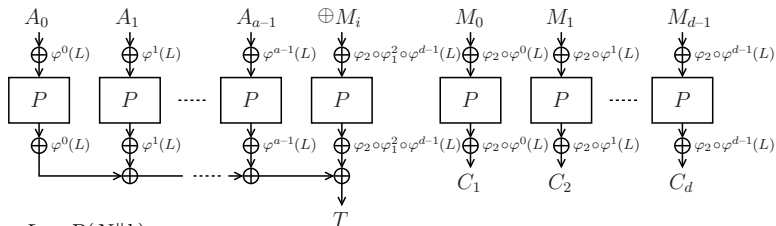
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# Application to AE: OPP

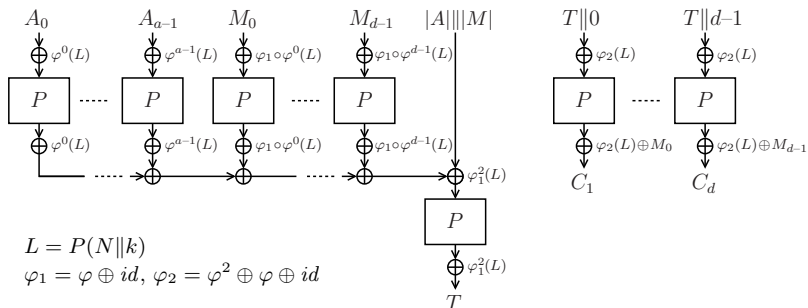


$$L = P(N||k)$$

$$\varphi_1 = \varphi \oplus id, \varphi_2 = \varphi^2 \oplus \varphi \oplus id$$

- Offset Public Permutation (OPP)
- Generalization of OCB3:
  - Permutation-based
  - More efficient MEM masking
- Security against nonce-respecting adversaries
- 0.55 cpb with reduced-round BLAKE2b

# Application to AE: MRO



- Misuse-Resistant OPP (MRO)
- Fully nonce-misuse resistant version of OPP
- 1.06 cpb with reduced-round BLAKE2b



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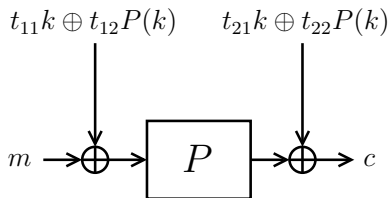
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# XPX

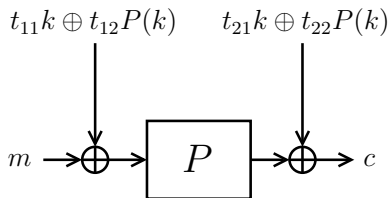
- XPX by Mennink [Men16]:



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# XPX

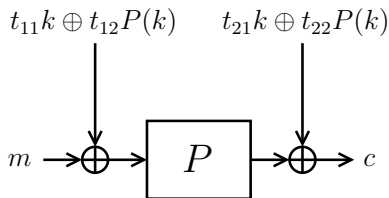
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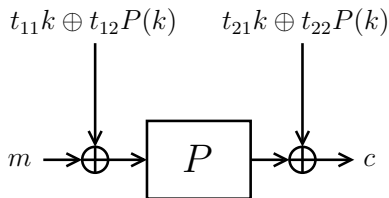
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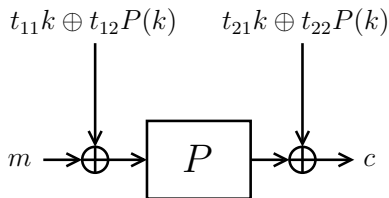
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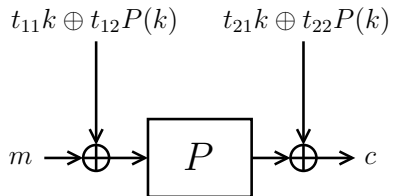
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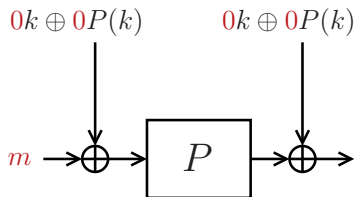


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## XPX: Weak Tweaks



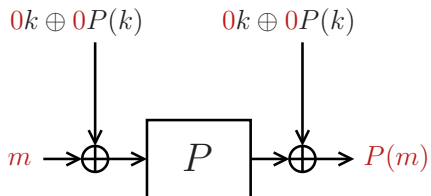
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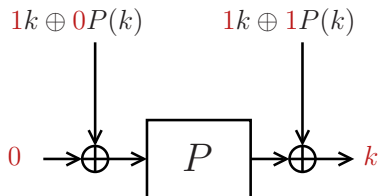


## XPX: Weak Tweaks



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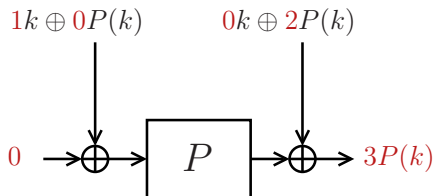
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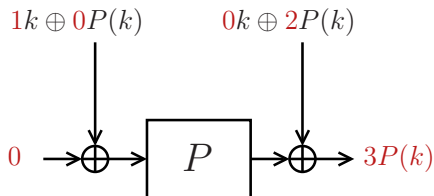


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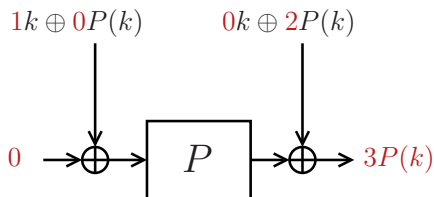
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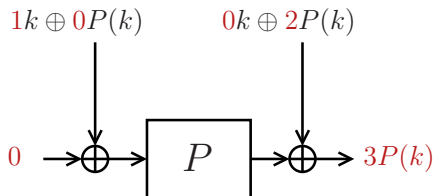
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### “Valid” Tweak Sets

- Technical definition to eliminate weak cases

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### “Valid” Tweak Sets

- Technical definition to eliminate weak cases
- $\mathcal{T}$  invalid  $\iff$  XPX insecure
- $\mathcal{T}$  valid  $\iff$  XPX single- or related-key secure

## XPX Covers Even-Mansour



for  $\mathcal{T} = \{(1, 0, 1, 0)\}$

## XPX Covers Even-Mansour



for  $\mathcal{T} = \{(1, 0, 1, 0)\}$

- Single-key STPRP secure (surprise?)



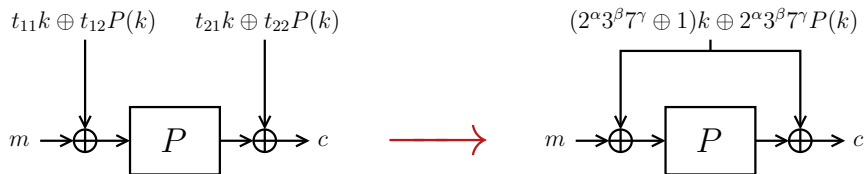
## XPX Covers Even-Mansour



for  $\mathcal{T} = \{(1, 0, 1, 0)\}$

- Single-key STPRP secure (surprise?)
- Generally, if  $|\mathcal{T}| = 1$ , XPX is a normal blockcipher

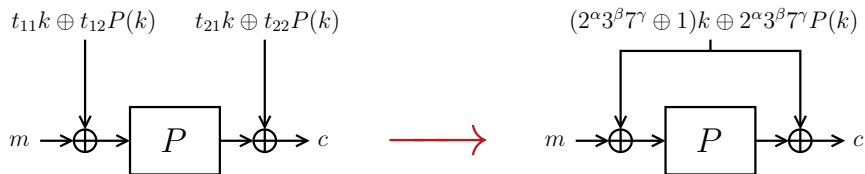
## XPX Covers XEX With Even-Mansour



$$\text{for } \mathcal{T} = \left\{ \begin{array}{l} (2^{\alpha}3^{\beta}7^{\gamma} \oplus 1, 2^{\alpha}3^{\beta}7^{\gamma}, \\ 2^{\alpha}3^{\beta}7^{\gamma} \oplus 1, 2^{\alpha}3^{\beta}7^{\gamma}) \end{array} \mid (\alpha, \beta, \gamma) \in \{\text{XEX-tweaks}\} \right\}$$

- $(\alpha, \beta, \gamma)$  is in fact the “real” tweak

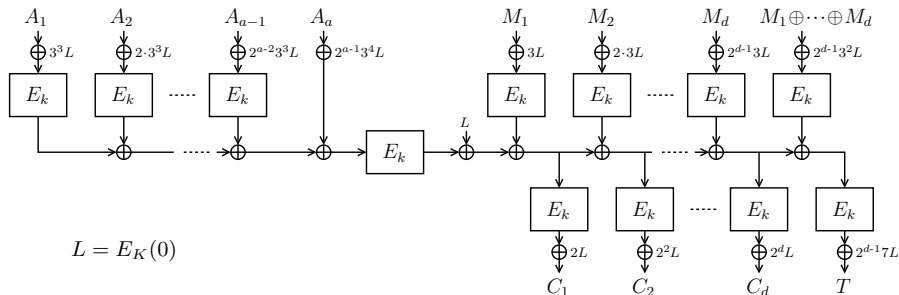
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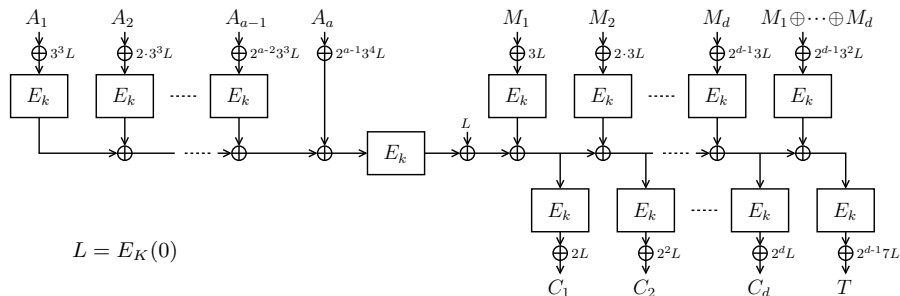
- $(\alpha, \beta, \gamma)$  is in fact the “real” tweak
- Related-key STPRP secure (if  $2^{\alpha}3^{\beta}7^{\gamma} \neq 1$ )

# Application to AE: COPA and Prøst-COPA



- By Andreeva et al. (2014)
- Implicitly based on XEX based on AES

## Application to AE: COPA and Prøst-COPA



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- Prøst-COPA by Kavun et al. (2014):  
COPA based on XEX based on Even-Mansour

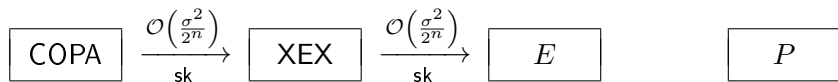
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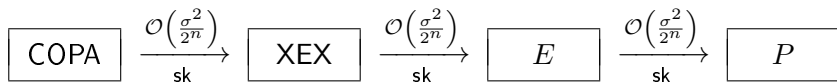
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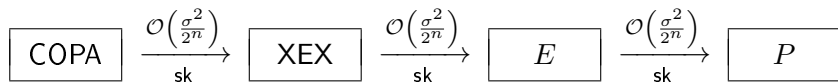
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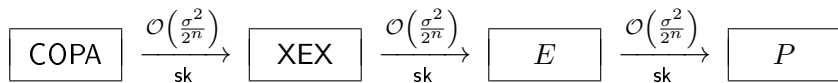
## Related-Key Security of COPA

- Existing proof generalizes



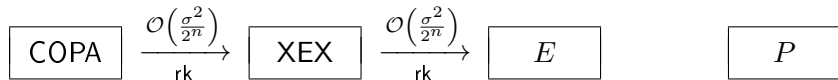
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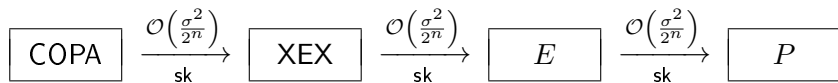
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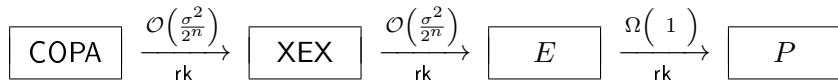
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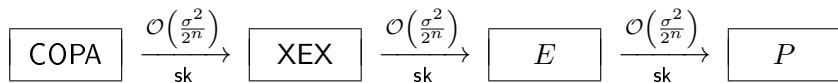
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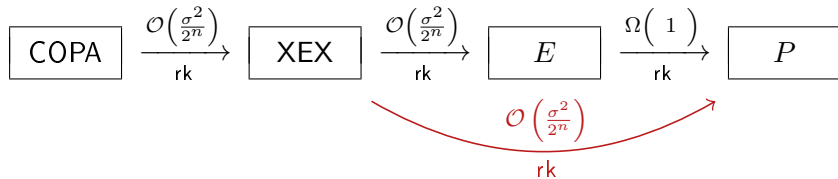
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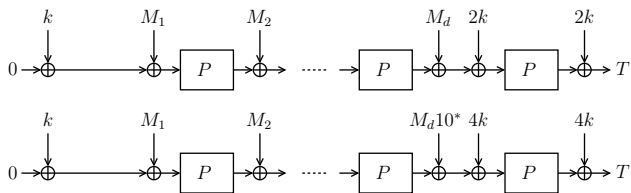


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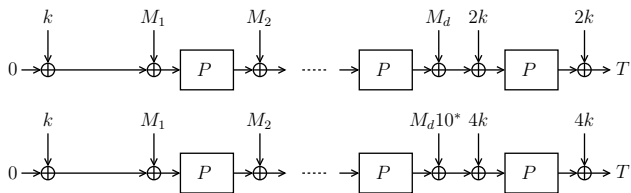
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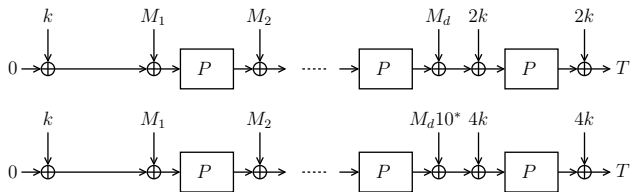
- Original proof based on 3 EM's: 
$$\begin{cases} E_k(m) = P(m \oplus k) \oplus k \\ E_k(m) = P(m \oplus 3k) \oplus 2k \\ E_k(m) = P(m \oplus 5k) \oplus 4k \end{cases}$$

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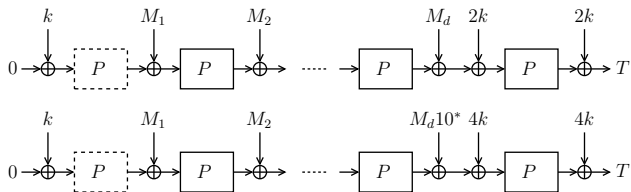
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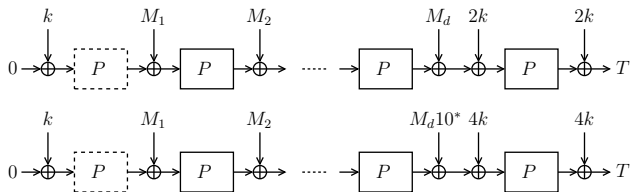
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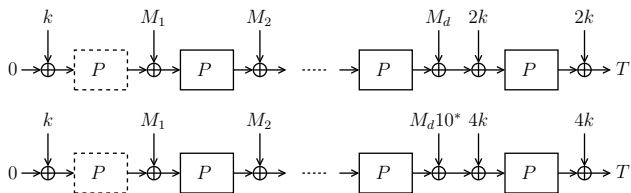


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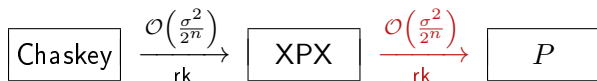


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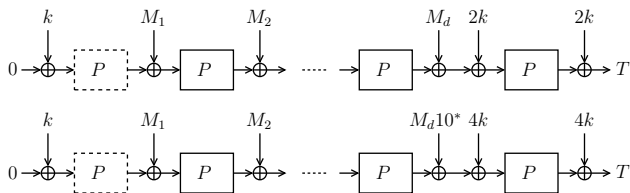
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- Approach can also be applied to:
  - Keyed Sponge and Duplex
  - 10 Sponge-inspired CAESAR candidates

# Outline

Dedicated Design

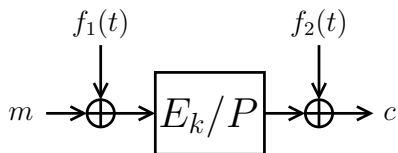
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Tweakable Blockciphers Based on Masking

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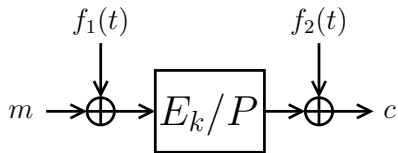
Conclusion

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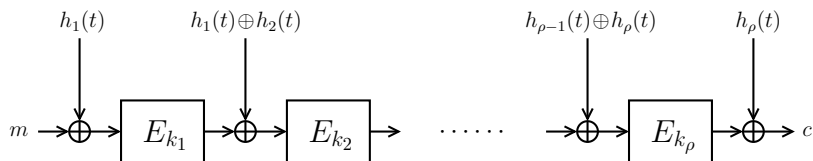
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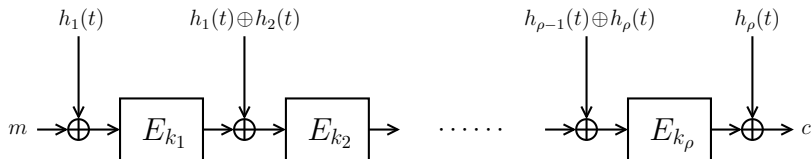
- “Birthday-bound”  $2^{n/2}$  security at best
- Overlying modes inherit security bound
- If  $n$  is large enough  $\rightarrow$  no problem
- If  $n$  is small  $\rightarrow$  “beyond birthday-bound” solutions
  - Cascading
  - Tweak-rekeying

## Cascading LRW's



- $\text{LRW}_2[\rho]$ : concatenation of  $\rho$   $\text{LRW}_2$ 's
- $k_1, \dots, k_\rho$  and  $h_1, \dots, h_\rho$  independent

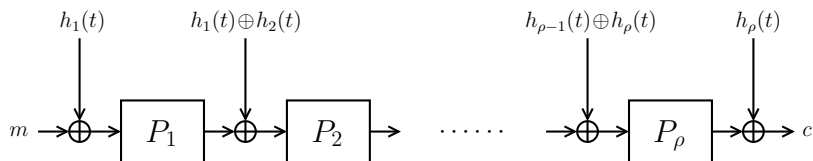
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- $\rho = 2$ : secure up to  $2^{2n/3}$  queries [LST12, Pro14]
- $\rho \geq 2$  even: secure up to  $2^{\rho n / (\rho + 2)}$  queries [LS13]
- Conjecture: optimal  $2^{\rho n / (\rho + 1)}$  security

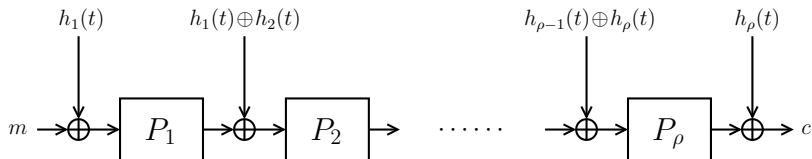


## Cascading TEM's



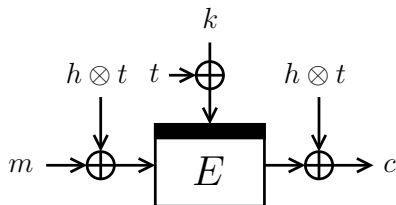
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# Cascading TEM's



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## Tweak-Rekeying



- Mingling tweak into **both key and state** works
- Secure up to  $2^n$  queries (in ICM!)
- Alternative constructions exist [Min09, Men15, WGZ+16]

More on “beyond birthday-bound security” on Thursday

# Outline

Dedicated Design

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## Tweakable Blockciphers: Simple and Powerful

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## Tweakable Blockciphers: Simple and Powerful

- Myriad applications to AE, MAC, encryption, . . .
- Choice of masking influences **efficiency and security**

## Security Level

- Birthday-bound security: okay if  $n$  is large enough
  - Permutation-based tweakable blockciphers
- Beyond birthday-bound security possible
  - More on Thursday

**Thank you for your attention!**

# SUPPORTING SLIDES

## MEM: Implementation

- State size  $b = 1024$
- LFSR on 16 words of 64 bits:

$$\varphi(x_0, \dots, x_{15}) = (x_1, \dots, x_{15}, (x_0 \lll 53) \oplus (x_5 \ll 13))$$

- $P$ : BLAKE2b permutation with 4 or 6 rounds



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- Main implementation results:

Platform	nonce-respecting				misuse-resistant	
	AES-GCM	OCB3	Deoxys <sup>≠</sup>	OPP <sub>4</sub>	OPP <sub>6</sub>	
Cortex-A8	38.6	28.9	-	4.26	5.91	
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Cortex-A8	38.6	28.9	-	4.26	5.91	-	-	8.07	11.32
Sandy Bridge	2.55	0.98	1.29	1.24	1.91	-	≈ 2.58	2.41	3.58
Haswell	1.03	0.69	0.96	0.55	0.75	1.17	≈ 1.92	1.06	1.39

## MEM: Parallelizability

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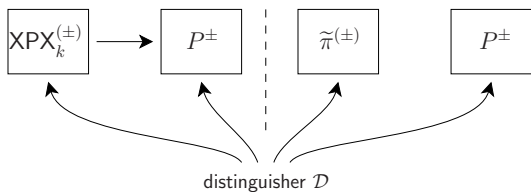
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- Parallelizable (AVX2) and word-sliceable

# XPX: Single-Key Security

## (Strong) Tweakable PRP

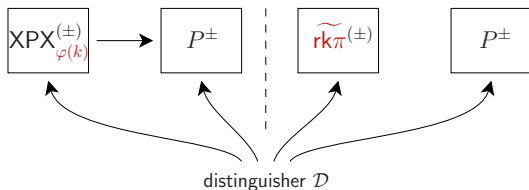


- Information-theoretic indistinguishability
  - $\tilde{\pi}$  ideal tweakable permutation
  - $P$  ideal permutation
  - $k$  secret key

$\mathcal{T}$  is valid  $\implies$  XPX is (S)TPRP up to  $\mathcal{O}\left(\frac{q^2 + qr}{2^n}\right)$

# XPX: Related-Key Security

## Related-Key (Strong) Tweakable PRP



- Information-theoretic indistinguishability
  - $\widetilde{rk\pi}$  ideal tweakable **related-key** permutation
  - $P$  ideal permutation
  - $k$  secret key
- $\mathcal{D}$  restricted to some set of **key-deriving functions**  $\Phi$

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## Key-Deriving Functions

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## Results

if $\mathcal{T}$ is valid, and for all tweaks:	security	$\Phi$
$t_{12} \neq 0$	TPRP	$\Phi_{\oplus}$
$t_{12}, t_{22} \neq 0$ and $(t_{21}, t_{22}) \neq (0, 1)$	STPRP	$\Phi_{\oplus}$

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$t_{11}, t_{12} \neq 0$	TPRP	$\Phi_{P\oplus}$
$t_{11}, t_{12}, t_{21}, t_{22} \neq 0$	STPRP	$\Phi_{P\oplus}$



# XPX: Security Proof Techniques

## Patarin's H-coefficient Technique

- Each conversation defines a transcript
- Define **good** and **bad** transcripts

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$$\mathbf{Adv}_{\text{XPX}}^{\text{rk-(s)prp}}(\mathcal{D}) \leq \varepsilon + \mathbf{Pr} \left[ \text{bad transcript for } (\widetilde{\text{rk}\pi}, P) \right]$$

↑ prob. ratio for **good** transcripts

# XPX: Security Proof Techniques

## Patarin's H-coefficient Technique

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↑ prob. ratio for **good** transcripts

- Trade-off: define **bad** transcripts smartly!

# XPX: Security Proof Techniques

## Before the Interaction

- Reveal “dedicated” oracle queries

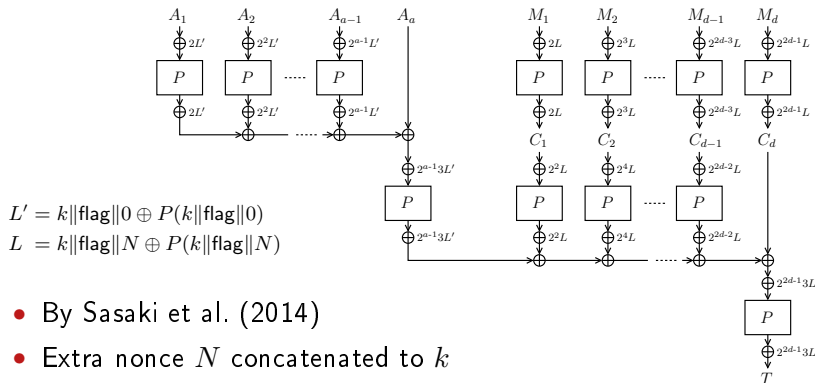
## After the Interaction

- Reveal key information
  - Single-key:  $k$  and  $P(k)$
  - $\Phi_{\oplus}$ -related-key:  $k$  and  $P(k \oplus \delta)$
  - $\Phi_{P\oplus}$ -related-key:  $k$  and  $P(k \oplus \delta)$  and  $P^{-1}(P(k) \oplus \varepsilon)$

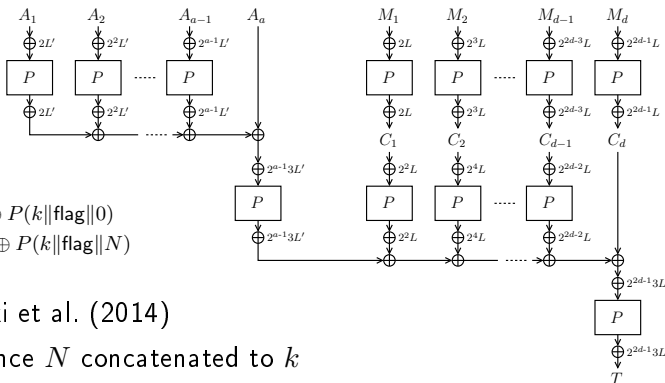
## Bounding the Advantage

- Smart definition of **bad** transcripts

# XPX: Application to AE: Minalpher



# XPX: Application to AE: Minalpher

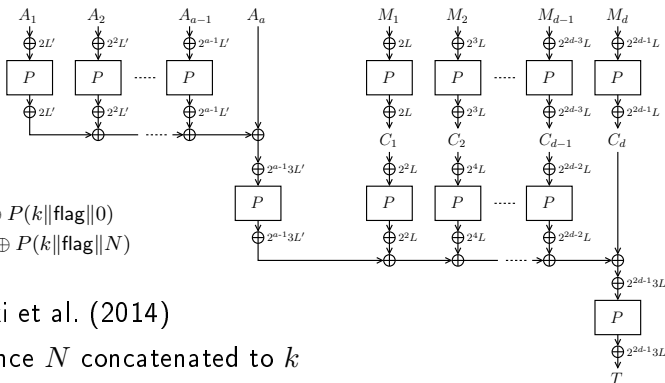


$$L' = k \parallel \text{flag} \parallel 0 \oplus P(k \parallel \text{flag} \parallel 0)$$

$$L = k \parallel \text{flag} \parallel N \oplus P(k \parallel \text{flag} \parallel N)$$

- By Sasaki et al. (2014)
- Extra nonce  $N$  concatenated to  $k$
- Based on XPX with  $\mathcal{T} = \{(2^\alpha 3^\beta, 2^\alpha 3^\beta, 2^\alpha 3^\beta, 2^\alpha 3^\beta)\}$

# XPX: Application to AE: Minalpher



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