Provable Security of BLAKE with Non-Ideal Compression Function

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Selected Areas in Cryptography
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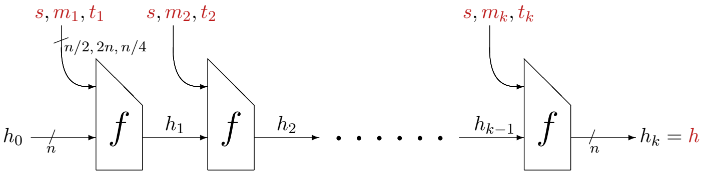
BLAKE

$$\mathcal{H} : \{0,1\}^{n/2} \times \{0,1\}^* \to \{0,1\}^n$$
$$\mathcal{H}(s, M) = h$$

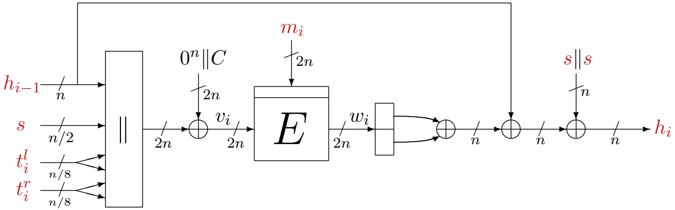
- SHA-3 finalist
- HAIFA design
- $m_1, \ldots, m_k$ padded message blocks of $2n$ bits
- $t_1, \ldots, t_k$ HAIFA-counter blocks of $n/4$ bits



BLAKE

$$f : \{0,1\}^n \times \{0,1\}^{n/2} \times \{0,1\}^{2n} \times \{0,1\}^{n/4} \rightarrow \{0,1\}^n$$
$$f(h_{i-1}, s, m_i, t_i) = h_i$$

- Local wide-pipe design
- f uses $E : \{0,1\}^{2n} \times \{0,1\}^{2n} \rightarrow \{0,1\}^{2n}$



State of the Art

pre f	sec f	col f	pre $\mathcal{H}$	sec $\mathcal{H}$	col $\mathcal{H}$	indiff $\mathcal{H}$
			2^n f ideal	2^n f ideal	$2^{n/2}$ f ideal	$2^{n/2}$ f ideal

- BLAKE follows HAIFA design:
  - $\rightarrow$ pre/sec/col/indiff security for f ideal

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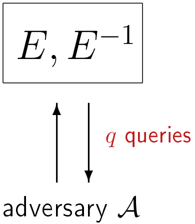
Analysis of BLAKE's $\mathcal{H}$ and f with underlying E ideal

Ideal Model Security: Col/Sec/Pre Resistance



- Ideal cipher model: $E : \{0,1\}^{2n} \times \{0,1\}^{2n} \to \{0,1\}^{2n}$
- $\mathcal{A}$ has query access to E

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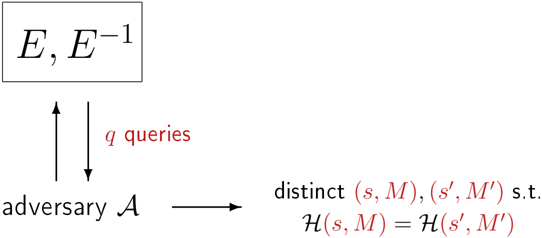

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(similar definitions if $\mathcal{A}$ attacks f)

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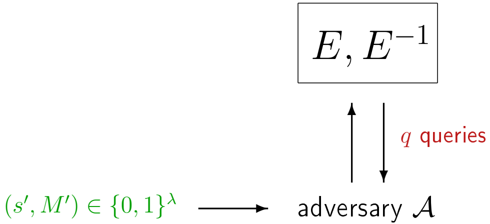

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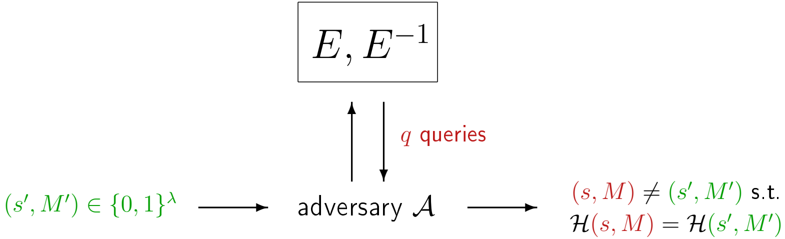

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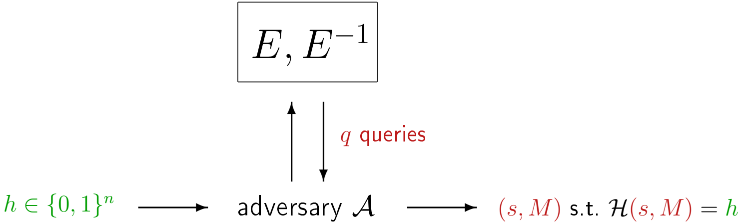

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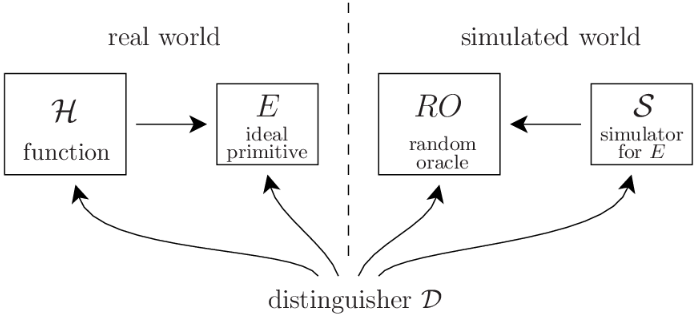
Ideal Model Security: Indifferentiability



- Indifferentiability of $\mathcal{H}$ from a random oracle
- $\mathcal{H}^E$ is indifferentiable from RO if $\exists$ simulator S such that $(\mathcal{H}, E)$ and $(RO, \mathcal{S})$ indistinguishable

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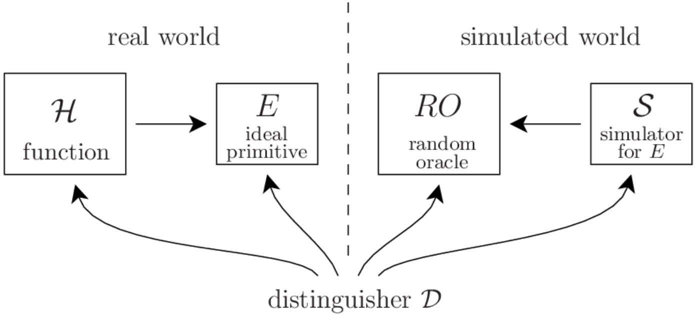
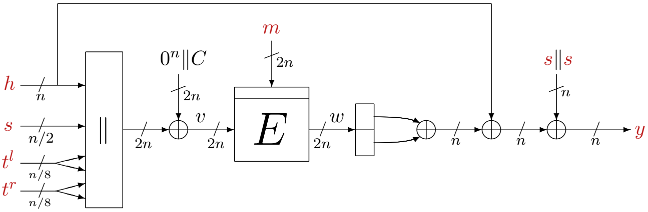


- Indifferentiability of $\mathcal{H}$ from a random oracle
- $\mathcal{H}^E$ is indifferentiable from RO if $\exists$ simulator S such that $(\mathcal{H}, E)$ and $(RO, \mathcal{S})$ indistinguishable
- Extension of indistinguishability: $\mathcal{D}$ may know structure of $\mathcal{H}$

(similar definitions if $\mathcal{D}$ attacks f)

Differentiability Attack f



f differentiable from RO in $2^{n/4}$ queries

- Differentiability: construct a distinguisher that tricks any simulator

(independently found by [Chang+11])

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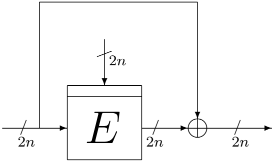

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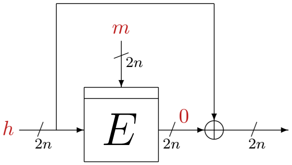

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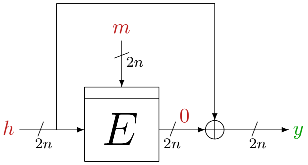

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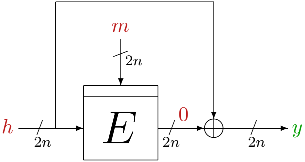

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$\mathcal{D}$ queries $E^{-1}(m, 0) \to h$
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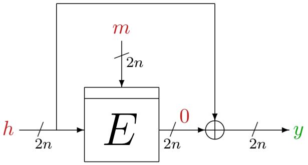

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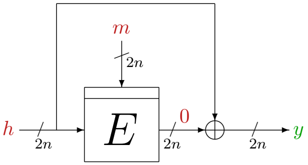

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Simulated world
$\mathcal{D}$ queries $\mathcal{S}^{-1}(m, 0) \to h$
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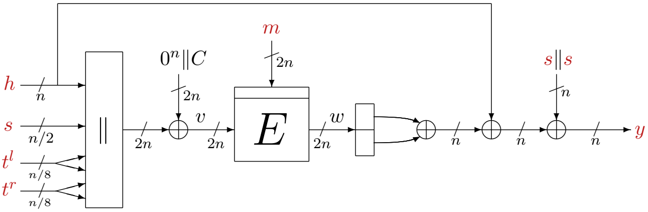
Differentiability Attack f



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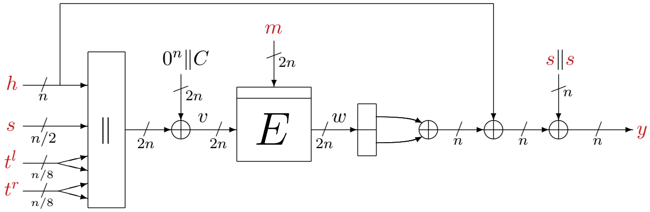

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- BLAKE's f: duplicate counter prevents this attack
  - $\mathcal{S}^{-1}$-responses non-compliant with duplicate counter are useless to $\mathcal{D}$
  - After $2^{n/4}$ queries, this gets suspicious

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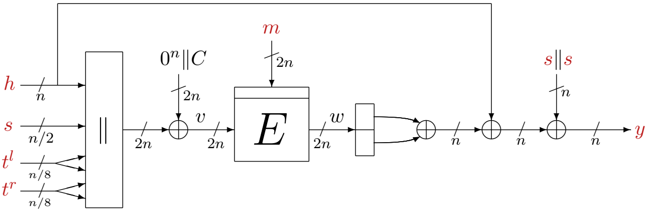

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- Invalidates assumption "f ideal"

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Differentiability attack on f

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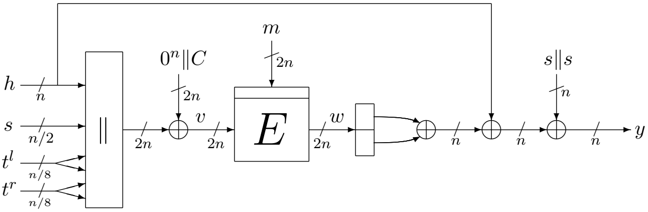
Preimage and Collision Resistance of BLAKE



$$\mathbf{Adv}_{\mathcal{H}}^{\mathrm{epre}}(q) \leq \mathbf{Adv}_{f}^{\mathrm{epre}}(q) = O(q/2^n)$$

- BLAKE preserves "epre"

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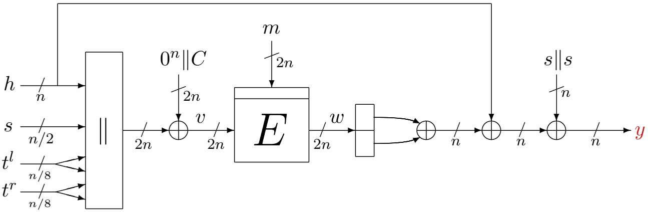

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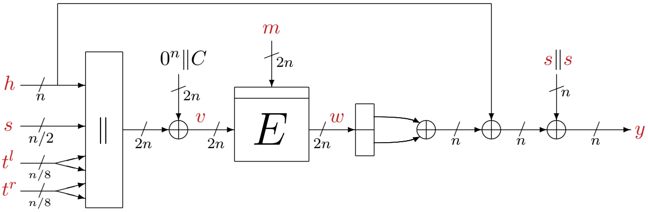

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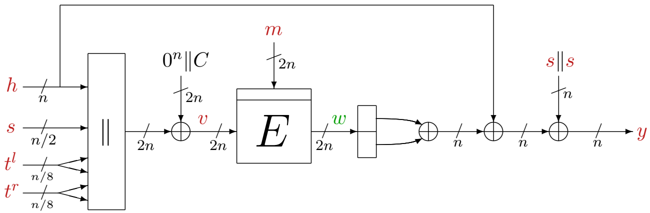

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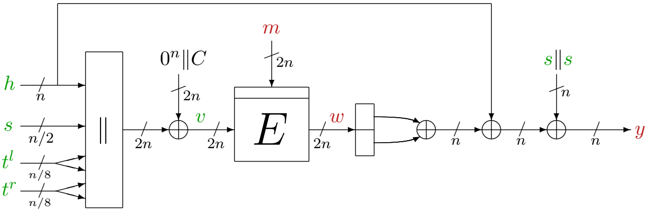

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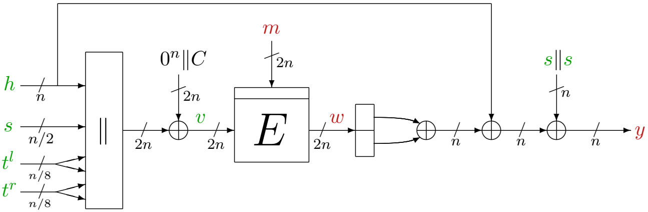

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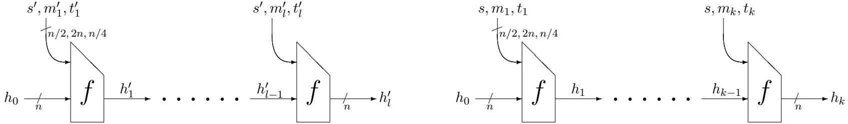
- Similarly, $\mathbf{Adv}_{\mathcal{H}}^{\mathrm{col}}(q) \leq \mathbf{Adv}_{f}^{\mathrm{col}}(q) = O(q^2/2^n)$

Second Preimage Resistance of BLAKE



$$\mathbf{Adv}_{\mathcal{H}}^{\text{esec}[\lambda]}(q) = O(q/2^n)$$

- "esec" not preserved: $\mathbf{Adv}_{\mathcal{H}}^{\text{esec}[\lambda]}(q) \not\lesssim \mathbf{Adv}_{f}^{\text{esec}[\lambda]}(q)$!

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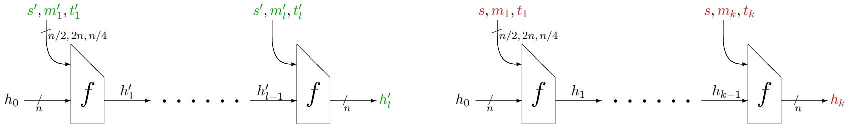

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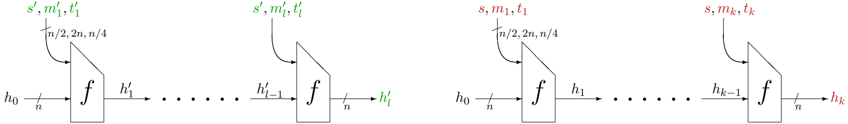

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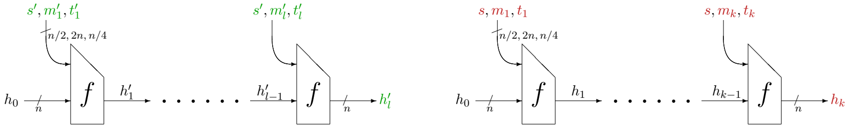

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- BLAKE achieves better second preimage resistance!
  $\rightarrow$ t_i fixes particular target state value from $\{h'_1, \ldots, h'_l\}$

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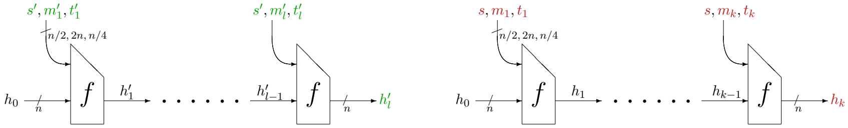

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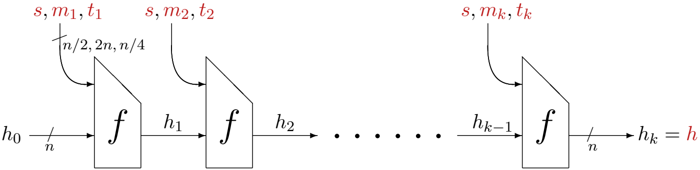
Indifferentiability of BLAKE



$$\mathbf{Adv}_{\mathcal{H}}^{\text{indiff}}(\mathcal{D}) = O((Kq)^2/2^n)$$

(where $\mathcal{D}$ makes at most q queries of length at most K blocks)

- We restore old indifferentiability bound of BLAKE in ICM
- High-level proof idea
  - $\mathcal{S}$ maintains graph: edges correspond to f-evaluations
  - Complete paths should be in correspondence with RO
- Technical details in paper

(independently found by [Chang+11])

Conclusions

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Differentiability attack on f

pre f	sec f	col f	pre $\mathcal{H}$	sec $\mathcal{H}$	col $\mathcal{H}$	indiff $\mathcal{H}$

Conclusions

pre f	sec f	col f	pre $\mathcal{H}$	sec $\mathcal{H}$	col $\mathcal{H}$	indiff $\mathcal{H}$
			2^n	2^n	$2^{n/2}$	$2^{n/2}$
			f ideal	f ideal	f ideal	f ideal

Differentiability attack on f

pre f	sec f	col f	pre $\mathcal{H}$	sec $\mathcal{H}$	col $\mathcal{H}$	indiff $\mathcal{H}$

Fix in ideal cipher model

pre f	sec f	col f	pre $\mathcal{H}$	sec $\mathcal{H}$	col $\mathcal{H}$	indiff $\mathcal{H}$
2^n		$2^{n/2}$	2^n	2^n	$2^{n/2}$	$2^{n/2}$
E ideal		E ideal	E ideal	E ideal	E ideal	E ideal

Comparison of SHA-3 Finalists [AMPS12]

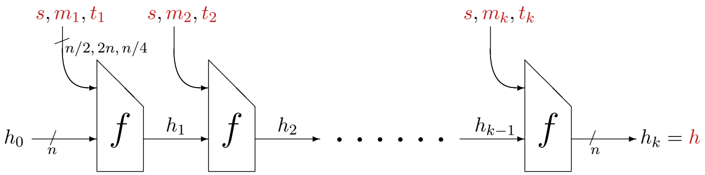
	l	m	pre	sec	col	indiff	assumption
BLAKE-256	256	512	256	256	128	128	E ideal
Grøstl-256	512	512	256	256–L	128	128	P, Q ideal
JH-256	1024	512	256	256	128	256	P ideal
Keccak-256	1600	1088	256	256	128	256	P ideal
Skein-256	512	512	256	256	128	256	E ideal
NIST's requirements			256	256–L	128	—	

	l	m	pre	sec	col	indiff	assumption
BLAKE-512	512	1024	512	512	256	256	E ideal
Grøstl-512	1024	1024	512	512–L	256	256	P, Q ideal
JH-512	1024	512	256	256	256	256	P ideal
Keccak-512	1600	576	512	512	256	512	P ideal
Skein-512	512	512	512	512	256	256	E ideal
NIST's requirements			512	512–L	256	—	

Supporting Slides

SUPPORTING SLIDES

Indifferentiability of BLAKE



$$\mathbf{Adv}_{\mathcal{H}}^{\mathrm{indiff}}(\mathcal{D}) = O((Kq)^2/2^n)$$

(where $\mathcal{D}$ makes at most q queries of length at most K blocks)

- Indifferentiability: construct a simulator that tricks any distinguisher
- $\mathcal{S}$ maintains graph: edges correspond to f-evaluations
  - Any $\mathcal{S}$-query defines at most one edge $h \xrightarrow{s\|m\|t} h'$
- Complete path: $h_0 \xrightarrow{s\|m_1\|t_1} h_1 \cdots \xrightarrow{s\|m_k\|t_k} h_k$ for correctly padded $(m_1, \ldots, m_k)$, $(t_1, \ldots, t_k)$

Indifferentiability of BLAKE

Forward Query $\mathcal{S}(m, v)$
if new query creates complete path **then**
(new query likely results in at most 1 complete path)
generate w in accordance with RO
else
generate w uniformly at random
end if
add new edge to graph

Inverse Query $\mathcal{S}^{-1}(m, w)$
(new query likely results in no complete path)
generate v uniformly at random
add new edge to graph