Formal Reasoning 2015
Exam
(29/01/15)

Before you read on, write your name, student number and study on the answer sheet! This exam consists of fifteen exercises (three exercises for each block of the course notes) and each of these exercises is worth six points. The mark for this test is the number of points divided by ten. The first ten points are free. Good luck!

1. Give a formula in the propositional logic that resembles the meaning of this sentence as well as possible: (6 points)

   If it rains I get wet, and it rains, but still I don't get wet!

   Use the following dictionary:

R	it rains
W	I get wet

2. This exercise is about the formula

   $$a \to \neg b \leftrightarrow \neg b \to a$$

   (a) Write this formula using parentheses according to the official grammar in the course notes. (3 points)

   (b) Give the truth table of this formula. (3 points)

3. This exercise is about the statement (6 points)

   $$f \to \neg g \equiv g \to \neg f$$

   where f and g are arbitrary formulas of the propositional logic.

   (a) Explain what this statement expresses. (3 points)

   (b) Does this statement hold if we take $f = a$ and $g = b$? (3 points)

4. Give a formula in the predicate logic that resembles the meaning of this sentence as well as possible: (6 points)

   When a man loves women, then there is a woman who loves that man.

   Use the dictionary:

M	the domain of men
W	the domain of women
$L(x, y)$	x loves y

   Interpret this sentence as a statement about all men.

5. Give a formula in the predicate logic with equality that resembles the meaning of this sentence as well as possible: (6 points)

   There exists exactly one woman that all men love.

   Use the dictionary from exercise 4.

6. Give an interpretation I_6 in a model M_6 for which the following formula of the predicate logic with equality holds: (6 points)

$$\forall x \in D\, \exists y \in D\, (x \neq y \wedge \forall z \in D\, (R(x,z) \leftrightarrow z = y))$$

   Explain your answer.

7. Give a language L_7 with alphabet $\Sigma = \{a, b\}$ for which (6 points)

$$L_7{}^* \cap \overline{L_7}^* \neq \{\lambda\}$$

   Explain your answer.

8. Give a finite automaton with a minimal number of states that recognizes the language (6 points)

$$L_8 := \mathcal{L}((aa \cup bb \cup (ab \cup ba)(aa \cup bb)^*(ab \cup ba))^*)$$

9. Consider the following context-free grammar G_9:

$$S \to AS \mid \lambda$$
$$A \to ab$$

   (a) Is G_9 right-linear? Explain your answer. (1 punt)

   (b) Is $\mathcal{L}(G_9)$ regular? Explain your answer. (1 punt)

   (c) Give an invariant that proves that (4 points)

$$ba \notin \mathcal{L}(G_9)$$

   Explain your answer.

10. Define a tree with 5775 vertices in which the length of the longest path is 2015. Write your answer as a pair $\langle P_{10}, L_{10} \rangle$. (6 points)

11. This exercise is about the recurrence relations:

$$f(m,0) = 0$$
$$f(m,n+1) = f(m,n) + m$$

(a) Show how $f(3,3)$ is computed using these recurrence relations. (2 points)

(b) Prove with induction on n that (4 points)

$$f(m,n) = m \cdot n$$

for all $m, n \geq 0$.

12. Jan, Piet and Klaas want to divide six different objects among each other in such a way that everybody gets two objects. In how many ways can they do this? Explain how you calculated this and which binomial coefficients you have used for this. (6 points)

13. Give a formula in the epistemic logic that formalizes the following sentence as well as possible: (6 points)

I know that I won't get wet when it rains, because if I know that it rains I bring an umbrella and if I bring an umbrella I know that I won't get wet.

Use as dictionary:

R	it rains
W	I get wet
U	I bring an umbrella

14. Give a serial Kripke model $\mathcal{M}_{14}$ in which the formula of the modal logic $(\Box a \to \Box b) \to \Box(a \to b)$ is not true. Explain your answer. (6 points)

15. Give an LTL formula that expresses that a always becomes true again and again, but that between the moments where a is true, always b needs to be true at least once. (6 points)