

Formal Reasoning 2014
Test Block 4: Discrete mathematics
(2/12/14)

Before you read on, write your name, student number and study on the answer sheet! The mark for this test is the number of points divided by ten. The first ten points are free. This test is 'closed book' and you are not allowed to use an electronic calculator. Good luck!

1. We define the graph

$$G_1 = (\{x \mid x \text{ is a province of the Netherlands}\}, \\ \{(x, y) \mid x \text{ shares a border with } y\})$$

(see the map in the appendix) This graph has 12 vertices and 23 edges.

- (a) Draw the graph G_1 on the appendix. (Place the vertices into the provinces.) (10 points)
 - (b) Give the chromatic number of G_1 . Explain your answer. (10 points)
 - (c) Does this graph contain a subgraph isomorphic to $K_{2,2}$? Explain your answer. (10 points)
2. Give a planar bipartite graph that does have an Euler circuit, but does not have a Hamilton path. Explain your answer. (10 points)
3. We define a sequence of numbers $(a_n)_{n \in \mathbb{N}}$ via these recurrence relations:

$$a_0 = 1 \\ a_{n+1} = 3a_n - 1 \quad \text{for } n \geq 0$$

- (a) Compute the value a_6 (without using the formula below). Explain how you derived this value. (15 points)
- (b) Prove by induction that

$$a_n = \frac{1}{2}(3^n + 1)$$

holds for all $n \geq 0$. (15 points)

4. A group of twelve students want to play volleyball. In how many ways can they divide this group into two teams of six players each? Explain how you computed your answer. (20 points)

Appendix Test 4 Formal Reasoning

Name: _____ Student number: _____ Study: _____



Instead of using the full names of the provinces in the graph, use the abbreviations *DR*, *FL*, *FR*, *GE*, *GR*, *LI*, *NB*, *NH*, *OV*, *UT*, *ZE*, *ZH* or the numbers 1 till 12 as vertices.