

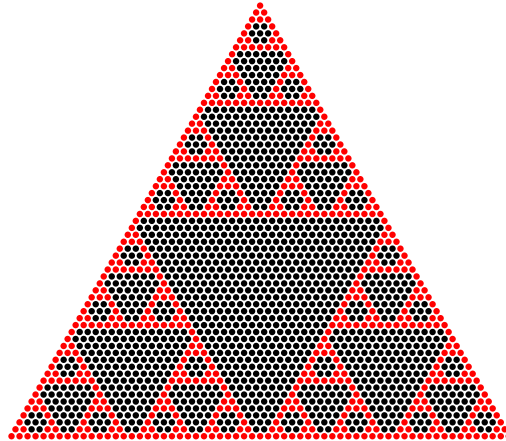
Formal Reasoning 2015
Additional Test
(13/01/16)

Before you read on, write your name, student number and study on the answer sheet! Each exercise is worth 18 points. The mark for this test is the number of points divided by ten. The first ten points are free. The test is *closed book*. Good luck!

1. The operator $|$ named *Sheffer stroke* is defined in such a way that $f | g \equiv \neg(f \wedge g)$. Give a formula f_1 in the propositional logic that has the Sheffer stroke as its only connective (so in particular the use of \neg , \wedge , et cetera is prohibited) such that $f_1 \equiv a \vee b$.
2. Give a model in which the following formula of the predicate logic is true.

$$\begin{aligned}
& (\forall x \in D \exists y \in D \forall y' \in D [R(x, y') \leftrightarrow y' = y]) \wedge \\
& (\forall x_1, x_2, y \in D [R(x_1, y) \wedge R(x_2, y) \rightarrow x_1 = x_2]) \wedge \\
& (\exists z \in D \forall x \in D \neg R(x, z))
\end{aligned}$$

3. Give a finite automaton with a minimal number of states that recognizes the language $\mathcal{L}((a^*b)^*)$.
- 4.



Above we have 2016 dots, because $(63 \cdot 64)/2 = 2016$. Determine using recursion how many of these dots are red.

5. Give an LTL formula f_5 such that the only Kripke model of f_5 with $V(x_i) \subseteq \{a, b\}$ is the model where

$$V(x_i) = \begin{cases} \{a\} & \text{if } i \text{ is even} \\ \{b\} & \text{if } i \text{ is odd} \end{cases}$$