

Formal Reasoning 2016
Test Block 1: Propositional Logic
(07/09/16)

Before you read on, write your name, student number and study on the answer sheet!

The mark for this test is the number of points divided by ten. The first ten points are free. For each (sub)question you can score ten points. Good luck!

The first two exercises use the following ‘dictionary’:

RB there is a rainbow
 R it rains
 S the sun shines

1. Give formulas of propositional logic that give the meaning of the following two English sentences:

- (a) *There only is a rainbow if it rains and the sun shines.*
(b) *There is a rainbow because it rains and the sun shines.*

2. Give English sentences that describe the meaning of the following two formulas of propositional logic as well as possible:

- (a)

$$R \rightarrow S \rightarrow RB$$

- (b)

$$((S \rightarrow R) \rightarrow S) \rightarrow S$$

3. This exercise is concerned with the formula:

$$a \leftrightarrow \neg a \leftrightarrow a \leftrightarrow \neg a$$

- (a) Write this formula with parentheses according to the official grammar in the course notes.
(b) Give the truth table of this formula.
(c) Does $\models a \leftrightarrow \neg a \leftrightarrow a \leftrightarrow \neg a$ hold? Explain your answer.
4. (a) Is there a model v in which $a \rightarrow \neg b$ and $\neg b \rightarrow a$ are both false? If so, give such a model and explain why it has this property. If not, explain why such a model does not exist.
(b) Are there formulas f and g such that:

$$f \rightarrow \neg g \models \neg g \rightarrow f$$

If so, give formulas like that and explain why they have this property. If not, explain why formulas like that do not exist.