Formal Reasoning 2016
Test Block 4: Discrete Mathematics
(30/11/16)

Before you read on, write your name, student number and study on the answer sheet!

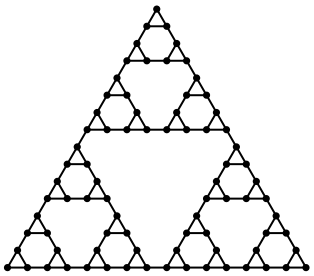
The mark for this test is the number of points divided by ten. The first ten points are free. For each (sub)question you can score ten points. Good luck!

1. Give a connected planar graph that has an Euler circuit, and in which not all vertices have degree two. Draw the graph in a planar representation, and explain why it has the required properties.

2. (a) Does there exist a tree that has a Hamilton path?
   (b) Does there exist a tree that does not have a Hamilton path?
   (c) Does there exist a tree that has a Hamilton circuit?

   Explain your answer for each of these three questions.

3. We define graphs G_n for $n \geq 1$, in which the vertices are the legal positions of the Towers of Hanoi with n disks, and the edges correspond to legal moves between those positions. For example the graph G_4 is:



   We write e_n for the number of edges in G_n. The sequence e_n satisfies the recursive equations:

$$e_1 = 3$$
$$e_{n+1} = 3e_n + 3$$

   (a) How many isomorphisms are there from G_1 to G_1?
   (b) Give a formula without recursion for the number of vertices in G_n.
   (c) Compute the number of edges in the graph G_4 using the recursive equations that were given above. Include your intermediate results.
   (d) Prove by induction that $e_n = \frac{1}{2}(3^{n+1} - 3)$ for all $n \geq 1$. In this proof you may use the recursive equations that were given above.

4. Calculate $(x+y)^7$ according to the binomial theorem, with the coefficients as explicit numbers. Indicate where the relevant binomial coefficients occur, both in your answer as well as in Pascal's triangle.