Formal Reasoning 2018
Test Block 3: Discrete Mathematics and Modal Logic
(19/12/18)

Before you read on, write your name, student number and study on the answer sheet!

We will only look at scratch paper if it has your name on it and you refer to it on the answer sheet. If not, we prefer that you do not hand in your scratch paper.

The mark for this test is the number of points divided by ten. The first ten points are free. For each (sub)question it is indicated how many points you can score. Good luck!

1. (a) Give a connected graph G_1 with a minimal number of vertices such that it has a Hamiltonian circuit, but does not have an Eulerian circuit. (10 points)

   (b) Give a connected graph G_1' with a minimal number of vertices such that it has an Eulerian circuit, but does not have a Hamiltonian circuit. (10 points)

   You do not need to explain why your graphs have the required properties.

2. The following equality holds: (20 points)

$$1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! = 23 = 4! - 1$$

   We want to show that this pattern holds in general. For this we define:

$$s_n := 1 \cdot 1! + \cdots + (n-1) \cdot (n-1)!$$

   For example $s_4 = 23$. This corresponds to the recursion equations:

$$\begin{aligned} s_2 &= 1 \\ s_{n+1} &= s_n + n \cdot n! \qquad \text{for } n \geq 2 \end{aligned}$$

   Prove from this with induction that $s_n = n! - 1$.

3. There are five rhyme schemes for a poem with three lines: AAA, AAB, ABA, ABB and ABC. Give the number of rhyme schemes for a poem with *four* lines, and show how this number relates to Bell numbers by giving a relevant part of an appropriate triangle of numbers. (20 points)

4. Suppose I lost my keys, but I am not aware of this, so I still think I know my keys are in my pocket. Then the following sentence will be true: (15 points)

   > *I do not know that it is not the case that I know my keys are in my pocket.*

   Give a formula of epistemic logic *without using negation signs*, that gives the meaning of this sentence. Use for a dictionary:

P	my keys are in my pocket

5. Give an LTL Kripke model $\mathcal{M}_5$ such that (15 points)

$$\mathcal{M}_5 \vDash (\mathcal{G}\mathcal{F}a) \wedge (\mathcal{G}\mathcal{F}\neg a) \wedge \mathcal{G}(a \leftrightarrow \mathcal{X}\mathcal{X}\mathcal{X}a)$$