Formal Reasoning 2019
Test Block 2: Languages and Automata
(06/11/19)

Before you read on, write your name, student number and study on the answer sheet!

We will only look at scratch paper if it has your name on it and you refer to it on the answer sheet. If not, we prefer that you do not hand in your scratch paper.

The mark for this test is the number of points divided by ten. The first ten points are free. Good luck!

1. The equality (15 points)
$$\overline{L \cup L'} = \overline{L} \cup \overline{L'}$$
does not hold for all languages L and L' over the alphabet $\{a, b\}$. Give languages L and L' over this alphabet for which this equality does not hold, and *explain* why this is the case.

2. (a) Give a deterministic finite automaton with alphabet $\{a, b, c\}$ for the language: (15 points)
$$L_2 := \{w \in \{a, b, c\}^* \mid \text{adjacent symbols in } w \text{ differ}\}$$
We have $abacb \in L_2$ because in $abacb$ all symbols differ from their predecessor, but $abba \notin L_2$, because there are two bs next to each other in $abba$. We also have $\lambda \in L_2$, because there are no adjacent symbols in λ at all.
Hint: Let the states of the automaton correspond to the last symbol that has been read thus far.

   (b) Give the right linear context-free grammar associated with the automaton from the previous sub-exercise. For this exercise it does not matter whether that automaton was correct for the language. (15 points)

3. Give a regular expression for the language: (15 points)
$$L_3 := \{w \in \{a, b, c\}^* \mid w \text{ does not contain } ab\}$$

4. Consider the context-free grammar G_4 : (15 points)
$$S \to A \mid bS$$
$$A \to aA \mid cS \mid \lambda$$
We want to show that $ab \notin \mathcal{L}(G_4)$ and are considering the predicate
$$P_4(w) := \big(w \text{ does not contain any of: } ab,\ aS,\ Ab,\ Sb,\ SS\big)$$
but this does not work. Explain why.

5. Explain why each language that can be recognized by a non-deterministic finite automaton, also can be recognized by a non-deterministic finite automaton that has a *single* final state. (15 points)