First exercise for Proof Assistants 2012

The exercise is to formalize a small proof using the Mizar proof assistant. The version of Mizar to be used is 7.12.01, of which the MML has version 4.166.1132. The statement to be proved is

$$x \neq 0 \ \wedge \ \left|\ln|x|\right| > 2 \ \wedge \ \int_0^{|x|} t\,dt \leq 1 \ \Rightarrow \ -\frac{1}{e^2} < x < \frac{1}{e^2}$$

Here is a possible proof:

Proof. We calculate

$$\int_0^{|x|} t\,dt = \left[\frac{1}{2}t^2\right]_0^{|x|} = \frac{1}{2}x^2$$

from which we deduce that

$$|x| \leq \sqrt{2}$$

We are also given that

$$\ln|x| < -2 \quad \text{or} \quad \ln|x| > 2$$

from which we get that

$$|x| < e^{-2} \quad \text{or} \quad |x| > e^2$$

But because we have

$$\sqrt{2} < e^2$$

the only possibility that remains is

$$|x| < \frac{1}{e^2}$$

This is trivially equivalent to the statement that needs to be proved. □

The *hard* deadline for this exercise is **June 20**. As a half-way checkpoint, **March 14** a version of the article has to be handed in in which the statement has been correctly formalized and in which only `*4` and `*1` errors remain.

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Here are some notes about this exercise.

- The exercise has two difficulties. The first difficulty is to calculate the integral. This needs mastery of the MML that is beyond the scope of this course. Therefore an article with two theorems already proved is given as a starting point:

  http://www.cs.ru.nl/~freek/courses/pa-2012/public/exercise.miz

The exercise can be made by adding Mizar text at the end of this article (and updating the environment accordingly).

(Of course, if you want to have a real challenge you can decide not to look at this file, and try to develop everything from scratch.)

- The second difficulty is the reasoning that gets from the inequalities about the natural logarithm to the inequalities about the exponential function. Useful lemmas for this might be:

```
FUNCT_1:35
LIMFUNC_1:def 3
SIN_COS:def 23
TAYLOR_1:9
TAYLOR_1:16
TAYLOR_1:18
XXREAL_1:235
```

- Articles that contain useful lemmas about the absolute value are:

```
ABSVALUE
COMPLEX1
SEQ_2
```

  Note that $|x|$ is in the MML both written as `abs(x)` and as `|.x.|`. The `synonym` that is used here is in the article `COMPLEX1`.

- Articles that contain useful lemmas about exponentiation are:

```
POWER
SQUARE_1
```

  Note that although the 'starting point' article given on the web site writes x^y as `x^y`, in the MML this is always written as `x to_power y`. The `synonym` that is used here is in the article `IRRAT_1`.