

Course: Type Theory and Coq

Exercises on Principal Types

All exercises are about Simple Type Theory à la Curry.

1. Determine the most general unifiers of
 - (a) $(\alpha \rightarrow \beta) \rightarrow \gamma$ and $\alpha \rightarrow \beta \rightarrow \gamma$
 - (b) $(\alpha \rightarrow \beta) \rightarrow \gamma$ and $\gamma \rightarrow \alpha \rightarrow \beta$
2. Compute the principal type of $\mathbf{S} := \lambda x. \lambda y. \lambda z. x z (y z)$.
3. Which of the following terms is typable? If it is, determine the *principal type*; if it isn't, show that the typing algorithm fails.
 - (a) $\lambda z x. z (x (\lambda y. y x))$
 - (b) $\lambda z x. z (x (\lambda y. y z))$
4. Compute the principal type of $M := \lambda x. \lambda y. x (y (\lambda z. x z z)) (y (\lambda z. x z z))$.
5. Which of the following terms is typable? If it is, determine the *principal type*; if it isn't, show that the typing algorithm fails.
 - $\lambda x. (\lambda y. x (x y)) (\lambda u v. u)$
 - $\lambda y. (\lambda x. x (x y)) (\lambda u v. u)$