$\lambda\!\rightarrow$

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reading

- Femke van Raamsdonk
  Logical Verification Course Notes
- Herman Geuvers
  Introduction to Type Theory
- Henk Barendregt
  Lambda Calculus with Types

  Cambridge University Press (to appear)

Curry-Howard

the Curry-Howard isomorphism

Haskell Curry
William Alvin Howard
Nicolaas Govert de Bruijn
Per Martin-Löf

logics $\underset{\text{isomorphic}}{\longleftrightarrow}$ *typed* lambda calculi
=
type theories

formulas $\longleftrightarrow$ types
proofs $\longleftrightarrow$ terms

logics versus type theories

many type theories

propositional logic	⟷	$\lambda{\to}$	
predicate logic	⟷	λP	*dependent types*
second order logic	⟷	$\lambda 2$	*polymorphism*

Martin-Löf's type theories

$\mathrm{MLW} \rightsquigarrow \mathrm{MLW}^{\mathrm{ext}}\mathrm{PU}_{<\omega}$

CC = calculus of constructions

'Coq logic' ⟷ pCIC = Coq's type theory

proof assistant

|

competitor of ZFC set theory

why types?

- semantics

$$\lambda x.xx$$
$$|$$
$$x \in \mathrm{dom}(x)?$$

- Curry-Howard isomorphism

logic!

- termination $=$ SN $=$ strong normalization

compute the value of any term

$\lambda\!\rightarrow$

names for $\lambda\!\to$

Alonzo Church

$\lambda\!\to$
Church's type theory
simply typed lambda calculus
simple type theory
simple theory of types
STT

description of a logic/type theory in general

1. syntax
    - terms
    - types
    - formulas
    - sequents $\rightsquigarrow$ $\Gamma \vdash A$
    - typing judgments $\rightsquigarrow$ $\Gamma \vdash M : A$
    - ...
2. rules
    - proof rules
    - typing rules
    - ...
3. *semantics*

description of $\lambda\!\rightarrow$

1. syntax
   - types
   - terms
   - contexts
   - judgments
2. rules
   - typing rules

types

grammar of types:

$$\underset{\substack{A, B, C, \ldots \\ \text{meta-variables for types}}}{A} \quad ::= \quad \underset{\substack{a, b, c, \ldots \\ \text{atomic types}}}{a} \mid \overset{\text{function types}}{(A \to A)}$$

$a \to b =$ type of functions from a to b

terms

grammar of pseudo-terms:

$$\underset{M, N, F, \ldots}{\underset{|}{M}} \quad ::= \quad \underset{\text{variable}}{\underset{|}{x}} \mid \overset{\text{function application}}{\overset{|}{(MM)}} \mid \underset{\text{abstraction}}{\underset{|}{(\lambda x : A.\, M)}}$$

$$\begin{array}{rl} \lambda x : A.\, M & \quad x \\ \lambda x^A\, M & \quad x^A \end{array}$$

explicitly typed variables = *Church-style*

Curry-style versus Church-style

- Curry-style:

$$\begin{array}{rcl} \lambda x.x & : & a \to a \\ \lambda x.x & : & b \to b \\ \lambda x.x & : & (a \to b) \to (a \to b) \end{array}$$

same term with multiple types

- Church-style:

$$\begin{array}{rcl} \lambda x : a.\, x & : & a \to a \\ \lambda x : b.\, x & : & b \to b \\ \lambda x : a \to b.\, x & : & (a \to b) \to (a \to b) \end{array}$$

different terms with each a single type

contexts

grammar of contexts:

$$\underset{\Gamma, \Delta, \ldots}{\underset{|}{\Gamma}} \quad ::= \quad \underset{\text{empty context}}{\underset{|}{\cdot}} \mid \Gamma, x : A$$

the $\cdot$ and possible following comma is not written:

$$x_1 : A_1, \ldots, x_n : A_n$$

judgments

typing judgments:

$$\underbrace{x_1 : A_1, \ldots, x_n : A_n}_{\Gamma} \vdash M : B$$

‘in context Γ the term M is well-typed and has type B’

terms and judgments: equivalence classes ‘up to alpha’

in rules: all x_i are different
Barendregt convention

typing rules

- variable rule

$$\frac{}{\Gamma \vdash x : A} \quad x : A \in \Gamma$$

- application rule

$$\frac{\Gamma \vdash F : A \to B \qquad \Gamma \vdash M : A}{\Gamma \vdash FM : B}$$

- abstraction rule

$$\frac{\Gamma, x : A \vdash M : B}{\Gamma \vdash \lambda x : A.\, M : A \to B}$$

example: typed K

untyped:

$$K \equiv \lambda xy.x$$

typed in $\lambda\!\rightarrow$:

$$K \equiv \lambda x : a.\ \underbrace{\lambda y : b.\, x}_{:\ b \rightarrow a} \ :\ a \rightarrow b \rightarrow a$$

example: type derivation for K

$$\cfrac{\cfrac{\cfrac{}{x : a,\, y : b \vdash x : a}}{x : a \vdash \lambda y : b.\, x \;:\; b \to a}}{\underbrace{}_{\Gamma \text{ is empty}} \vdash \lambda x : a.\, \underbrace{\lambda y : b.\, x}_{M} \;:\; a \to \underbrace{b \to a}_{B}}$$

minimal logic

minimal propositional logic

- *implicational* logic
  only connective is →
- intuitionistic
  not classical

$$\nvdash ((a \to b) \to a) \to a$$

logic styles:

1. Hilbert system
2. sequent calculus
3. natural deduction
   - Gentzen-style
   - Jaśkowsky/Fitch-style

formulas

grammar of formulas:

implication

$$A \quad ::= \quad a \mid (A \to A)$$

$A, B, C, \ldots$

meta-variables for formulas

$a, b, c, \ldots$

atomic propositions

proof rules

- implication introduction

$$\dfrac{\begin{array}{ccc} & \cancel{A} & \cancel{A} \\ \ddots & \vdots & \iddots \\ & B & \end{array}}{A \rightarrow B} \rightarrow I$$

- implication elimination
  = modus ponens

$$\dfrac{\begin{array}{cc} \vdots & \vdots \\ A \rightarrow B & A \end{array}}{B} \rightarrow E$$

example: proof of $a \longrightarrow b \longrightarrow a$

‘if a then it holds that if b then a’
‘a implies that b implies a’

$$\dfrac{\dfrac{\cancel{a}^{\,x}}{b \longrightarrow a}\,{\longrightarrow}I_y}{a \longrightarrow b \longrightarrow a}\,{\longrightarrow}I_x$$

proof terms

… and now in stereo!

logic

$$\cfrac{\cfrac{\cancel{a}^{x}}{b \longrightarrow a}\ {\longrightarrow}I_y}{a \longrightarrow b \longrightarrow a}\ {\longrightarrow}I_x$$

type theory

$$\cfrac{\cfrac{\cfrac{}{x : a,\ y : b \vdash x : a}}{x : a \vdash \lambda y : b.\, x \ :\ b \longrightarrow a}}{\vdash \lambda x : a.\, \lambda y : b.\, x \ :\ a \longrightarrow b \longrightarrow a}$$

BHK interpretation

Luitzen Egbertus Jan Brouwer
Arend Heyting
Andrey Kolmogorov

intuitionistic interpretation of logical connectives:

proof of	$A \wedge B$	$=$	pair of a proof of A and a proof of B
proof of	$A \vee B$	$=$	either a proof of A or a proof of B
proof of	$A \rightarrow B$	$=$	mapping of proofs of A to proofs of B
proof of	$\neg A$	$=$	proof of $A \rightarrow \bot$
proof of	$\bot$		does not exist
proof of	$\top$	$=$	the unique proof of $\top$

intuitionism

- classical logic

  $A \vee B$ = at least one of A and B holds
  $\exists x\, P(x)$ = there is an x for which $P(x)$ holds

  (but we might not be able to know which)

- intuitionistic logic = constructive logic

  $A \vee B$ = we can compute which of A or B holds
  $\exists x\, P(x)$ = we can compute an x for which $P(x)$ holds

$$\not\vdash\ a \vee \neg a$$
$$\not\vdash\ \neg\neg a \to a$$
$$\not\vdash\ ((a \to b) \to a) \to a$$

Luitzen Egbertus Jan Brouwer

is classical logic or intuitionistic logic more intuitive?

- classical:

$$ZF \vdash 2^{\aleph_0} = \aleph_1 \lor 2^{\aleph_0} \neq \aleph_1$$

$$ZF \nvdash 2^{\aleph_0} = \aleph_1$$

$$ZF \nvdash 2^{\aleph_0} \neq \aleph_1$$

- intuitionistic:

  M is a Turing machine that looks for a proof of $\bot$ in IZF

$$IZF \nvdash M{\downarrow} \lor \neg M{\downarrow}$$

$$IZF \vdash \neg\neg(M{\downarrow} \lor \neg M{\downarrow})$$

styles of logic

styles of logic

1. Hilbert system

   David Hilbert

2. sequent calculus

   Gerhard Gentzen

3. natural deduction
   - Gentzen-style

     Gerhard Gentzen

   - Jaśkowsky/Fitch-style

     Stanisław Jaśkowski
     Frederic Fitch

logic style 1: Hilbert system

- just one proof rule
  modus ponens

$$\begin{array}{cc} \vdots & \\ A & \\ \vdots & \\ A \to B & \\ \vdots & \\ B & \qquad \text{MP} \end{array}$$

- *axiom schemes*

$$\begin{array}{cc} A \to B \to A & \qquad \text{K} \\ (A \to B \to C) \to (A \to B) \to A \to C & \qquad \text{S} \end{array}$$

example: proof of $a \longrightarrow a$

1	$(a \longrightarrow (b \longrightarrow a) \longrightarrow a) \longrightarrow (a \longrightarrow b \longrightarrow a) \longrightarrow a \longrightarrow a$	S
2	$a \longrightarrow (b \longrightarrow a) \longrightarrow a$	K
3	$(a \longrightarrow b \longrightarrow a) \longrightarrow a \longrightarrow a$	MP 1,2
4	$a \longrightarrow b \longrightarrow a$	K
5	$a \longrightarrow a$	MP 3,4

Curry-Howard for Hilbert system

logic $\longleftrightarrow$ type theory

Hilbert system $\longleftrightarrow$ typed combinatory logic

proof of $a \to a$ $\longleftrightarrow$ $SKK =_\beta I$

deduction theorem $\longleftrightarrow$ converting lambda terms to combinatory logic

logic style 2: sequent calculus

sequents:

$$A_1, \ldots, A_n \vdash B_1, \ldots, B_m$$

to be read as:

$$A_1 \wedge \ldots \wedge A_n \rightarrow B_1 \vee \ldots \vee B_m$$

$A_1, \ldots, A_n$ and $B_1, \ldots, B_n$ are sets, not lists

intro/elim versus left/right

for each logical connective $\otimes$:

- natural deduction:

intro rules	$\otimes I$
elim rules	$\otimes E$

- sequent calculus:

left rules	$\otimes L$
right rules	$\otimes R$

proof rules

- assumption rule

$$\frac{}{\Gamma, A \vdash A, \Delta}\,\text{ass}$$

- left rule for implication

$$\frac{\Gamma \vdash A, \Delta \qquad \Gamma, B \vdash \Delta}{\Gamma,\ A \rightarrow B \vdash \Delta}\rightarrow\! L$$

- right rule for implication

$$\frac{\Gamma, A \vdash B, \Delta}{\Gamma \vdash A \rightarrow B,\ \Delta}\rightarrow\! R$$

example: proof of $a \longrightarrow b \longrightarrow a$

$$\dfrac{\dfrac{\dfrac{}{a, b \vdash a}\ \text{ass}}{a \vdash b \longrightarrow a}\ {\longrightarrow}R}{\vdash a \longrightarrow b \longrightarrow a}\ {\longrightarrow}R$$

cuts

- cut rule

$$\frac{\Gamma \vdash \Delta, A \qquad A, \Gamma \vdash \Delta}{\Gamma \vdash \Delta}\,\text{cut}$$

cut elimination theorem:

all provable statements can also be proved with a cut-free proof

general shape of sequent calculus proof rules

rules for $\vee$:

$$\frac{\Gamma, A \vdash \Delta \qquad \Gamma, B \vdash \Delta}{\Gamma,\ A \vee B \vdash \Delta}\,\vee L \qquad\qquad \frac{\Gamma \vdash A, B, \Delta}{\Gamma \vdash A \vee B,\ \Delta}\,\vee R$$

rules for $\otimes$:

$$\frac{\ldots}{\ldots\ \ \ldots \otimes \ldots\ \vdash \ldots}\,\otimes L \qquad\qquad \frac{\ldots}{\ldots\ \vdash\ \ldots \otimes \ldots\ \ \ldots}\,\otimes R$$

Curry-Howard for sequent calculus

Michel Parigot

$\lambda\mu$

Hugo Herbelin

$\bar{\lambda}\mu\tilde{\mu}$

Curry-Howard for *classical logic*

exceptions: throw/catch
variables for *continuations*

intuitionistic sequent calculus

- system LK: classical sequent calculus
- system LJ: intuitionistic sequent calculus

  only sequents with one formula on the right:

$$A_1, \ldots, A_n \vdash B$$

  proof rules adapted accordingly

logic style 3a: natural deduction, Gentzen-style

this system already has been presented
now in sequent presentation

instead of formulas:

$$B$$

now sequents:

$$\underbrace{A_1, \ldots, A_n}_{\text{open assumptions}} \vdash B$$

proof rules

- assumption rule

$$\frac{}{\Gamma \vdash A}\,\text{ass} \qquad A \in \Gamma$$

- implication introduction

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \to B}\to I$$

- implication elimination

$$\frac{\Gamma \vdash A \to B \qquad \Gamma \vdash A}{\Gamma \vdash B}\to E$$

example: proof of $a \longrightarrow b \longrightarrow a$

$$\dfrac{\dfrac{\dfrac{}{a, b \vdash a}\,\text{ass}}{a \vdash b \longrightarrow a}\,{\longrightarrow}I}{\vdash a \longrightarrow b \longrightarrow a}\,{\longrightarrow}I$$

general shape of natural deduction proof rules

rules for $\vee$:

$$\frac{\Gamma \vdash A}{\Gamma \vdash A \vee B}\,\vee I_l \qquad \frac{\Gamma \vdash B}{\Gamma \vdash A \vee B}\,\vee I_r \qquad \frac{\Gamma \vdash A \vee B \quad \Gamma, A \vdash C \quad \Gamma, B \vdash C}{C}\,\vee E$$

rules for $\otimes$:

$$\frac{\ldots}{\ldots \vdash \ldots \otimes \ldots}\,\otimes I \qquad \frac{\ldots \vdash \ldots \otimes \ldots \quad \ldots}{\ldots}\,\otimes E$$

intro/elim versus left/right, revisited

- natural deduction: introduction and elimination rules

$$\frac{\ldots \vdash \ldots}{\ldots \vdash \ldots \otimes \ldots}\otimes I \qquad \frac{\ldots \vdash \ldots \otimes \ldots}{\ldots \vdash \ldots}\otimes E$$

- sequent calculus: left and right rules

$$\frac{\ldots \vdash \ldots}{\ldots \otimes \ldots \vdash \ldots}\otimes L \qquad \frac{\ldots \vdash \ldots}{\ldots \vdash \ldots \otimes \ldots}\otimes R$$

is sequent calculus more attractive . . . ?

$$\frac{\Gamma, A, B \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta}\,\wedge L \qquad \frac{\Gamma \vdash A, \Delta \quad \Gamma \vdash B, \Delta}{\Gamma \vdash A \wedge B, \Delta}\,\wedge R$$

$$\frac{\Gamma, A \vdash \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \vee B \vdash \Delta}\,\vee L \qquad \frac{\Gamma \vdash A, B, \Delta}{\Gamma \vdash A \vee B, \Delta}\,\vee R$$

$$\frac{\Gamma \vdash A, \Delta}{\Gamma, \neg A \vdash \Delta}\,\neg L \qquad \frac{\Gamma, A \vdash \Delta}{\Gamma \vdash \neg A, \Delta}\,\neg R$$

$$\frac{\Gamma \vdash \Delta}{\Gamma, \top \vdash \Delta}\,\top L \qquad \frac{}{\Gamma \vdash \top, \Delta}\,\top R$$

$$\frac{}{\Gamma, \bot \vdash \Delta}\,\bot L \qquad \frac{\Gamma \vdash \Delta}{\Gamma \vdash \bot, \Delta}\,\bot R$$

. . . or is natural deduction more attractive?

$$\frac{\Gamma \vdash A \qquad \Gamma \vdash B}{\Gamma,\ A \land B}\,\land I \qquad \frac{\Gamma \vdash A \land B}{\Gamma \vdash A}\,\land E_l \quad \frac{\Gamma \vdash A \land B}{\Gamma \vdash B}\,\land E_r$$

$$\frac{\Gamma \vdash A}{\Gamma \vdash A \lor B}\,\lor I_l \qquad \frac{\Gamma \vdash B}{\Gamma \vdash A \lor B}\,\lor I_r \qquad \frac{\Gamma \vdash A \lor B \quad \Gamma, A \vdash C \quad \Gamma, B \vdash C}{C}\,\lor E$$

$$\frac{\Gamma, A \vdash \bot}{\Gamma \vdash \neg A}\,\neg I \qquad \frac{\Gamma \vdash \neg A \qquad \Gamma \vdash A}{\Gamma \vdash \bot}\,\neg E$$

$$\frac{}{\Gamma \vdash \top}\,\top I \qquad \frac{\Gamma \vdash \bot}{\Gamma \vdash A}\,\bot E$$

Curry-Howard for natural deduction, again

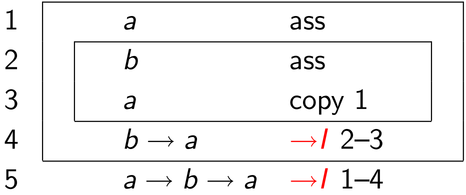
logic

$$\cfrac{\cfrac{\cfrac{}{a, b \vdash a}\,\text{ass}}{a \vdash b \longrightarrow a}\,{\longrightarrow}I}{\vdash a \longrightarrow b \longrightarrow a}\,{\longrightarrow}I$$

type theory

$$\cfrac{\cfrac{\cfrac{}{x : a,\ y : b \vdash x : a}}{x : a \vdash \lambda y : b.\, x \ : \ b \longrightarrow a}}{\vdash \lambda x : a.\, \lambda y : b.\, x \ : \ a \longrightarrow b \longrightarrow a}$$

logic style 3b: natural deduction, Jaśkowsky/Fitch-style



1	a	ass
2	b	ass
3	a	copy 1
4	$b \to a$	$\to I$ 2–3
5	$a \to b \to a$	$\to I$ 1–4

1	$a \vdash a$	ass
2	$a, b \vdash b$	ass
3	$a, b \vdash a$	weaken 1
4	$a \vdash b \to a$	$\to I$ 3
5	$\vdash a \to b \to a$	$\to I$ 4

detour elimination

detour elimination

detour = intro rule directly followed by corresponding elim rule

$$\dfrac{\dfrac{\vdots}{\ldots \otimes \ldots}\,\otimes I \qquad \ldots}{\ldots}\,\otimes E$$

detours for implication behave like cuts

sequent calculus: cut elimination
natural deduction: detour elimination

detour elimination theorem:

all provable statements can also be proved with a detour-free proof

example with a detour

$$\dfrac{\dfrac{\dfrac{\cancel{a}^{\,y}}{a \longrightarrow a}\ {\longrightarrow}I_y \qquad \cancel{a}^{\,x}}{a}\ {\longrightarrow}E}{a \longrightarrow a}\ {\longrightarrow}I_x$$

proof term:

$$\lambda x : a.\,(\lambda y : a.\,y)\,x$$

$$\lambda x.\,\underbrace{(\lambda y.y)\,x}_{\textit{redex}!}$$

. . . written with sequents and with proof terms

$$\dfrac{\dfrac{\dfrac{\dfrac{}{a,\ a \vdash a}\,\text{ass}}{a \vdash a \to a}\,{\to} I \qquad \dfrac{}{a \vdash a}\,\text{ass}}{a \vdash a}\,{\to}E}{\vdash a \to a}\,{\to}I$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{x : a,\ y : a \vdash y : a}}{x : a \vdash \lambda y : a.\, y \ :\ a \to a} \qquad \dfrac{}{x : a \vdash x : a}}{x : a \vdash (\lambda y : a.\, y)\, x \ :\ a}}{\vdash \lambda x : a.\, (\lambda y : a.\, y)\, x \ :\ a \to a}$$

detour elimination in general

$$\dfrac{\dfrac{\begin{array}{c}\cancel{A}\qquad\cancel{A}\\ \ddots\quad\vdots\quad\cdot^{\cdot^{\cdot}}\\ B\end{array}}{A \to B}\,{\to}I \qquad \begin{array}{c}\vdots\\ A\end{array}}{B}\,{\to}E \qquad\longrightarrow\qquad \begin{array}{c}\vdots\qquad\vdots\\ A\qquad A\\ \ddots\quad\vdots\quad\cdot^{\cdot^{\cdot}}\\ B\end{array}$$

proof of B using a *lemma* A

. . . written with sequents and with proof terms

$$\dfrac{\dfrac{\dfrac{\vdots}{\Gamma, A \vdash B}}{\Gamma \vdash A \to B}\,{\to}I \qquad \dfrac{\vdots}{\Gamma \vdash A}}{\Gamma \vdash B}\,{\to}E \quad \longrightarrow \quad \dfrac{\vdots}{\Gamma \vdash B}$$

$$\dfrac{\dfrac{\dfrac{\vdots}{\Gamma, x : A \vdash M : B}}{\Gamma \vdash \lambda x : A.\, M \ : \ A \to B}\,{\to}I \qquad \dfrac{\vdots}{\Gamma \vdash N : A}}{\Gamma \vdash (\lambda x : A.\, M)\; N \ : \ B}\,{\to}E \quad \longrightarrow \quad \dfrac{\vdots}{\Gamma \vdash M[x := N] \ : \ B}$$

normalization of proofs and terms

reduction corresponding to detour elimination:

$$(\lambda x : A.\, M)\, N \quad \rightarrow_{\beta} \quad M[x := N]$$

Curry-Howard:

logic $\longleftrightarrow$ type theory
proof $\longleftrightarrow$ term
detour elimination $\longleftrightarrow$ β-reduction
detour-free proof $\longleftrightarrow$ term in β-normal form

consistency

subject reduction

theorem (subject reduction = SR)
$\Gamma \vdash M : A$ and $M \twoheadrightarrow_\beta N$ then $\Gamma \vdash N : A$

proof: induction on the number of steps in the reduction

for a single step: induction on the definition of $\rightarrow_\beta$ using the subsitution lemma below $\square$

lemma (substitution lemma)
$\Gamma, x : B \vdash M : A$ and $\Gamma \vdash N : B$ then $\Gamma \vdash M[x := N] : A$

lemma (weakening)
$\Gamma \vdash M : A$ then $\Gamma, x : B \vdash M : A$

lemma (stengthening)
$\Gamma, x : B \vdash M : A$ and $x \notin FV(M)$ then $\Gamma \vdash M : A$

termination and confluence

theorem (strong normalization = SN)

$\Gamma \vdash M : A$ then there is no infinite $M \rightarrow_\beta M_1 \rightarrow_\beta M_2 \rightarrow_\beta \ldots$

proof later in the course

theorem

every inhabited type has an inhabitant in β-normal form

proof: combine subject reduction and strong normalization □

theorem (Church-Rosser = CR)

$$\begin{tikzcd}
& M \arrow[dl, two heads] \arrow[dr, two heads] & \\
M_1 \arrow[dr, dotted, two heads] & & M_2 \arrow[dl, dotted, two heads] \\
& N &
\end{tikzcd}$$

proof: the proof for the untyped case respects types □

long normal forms

if M has type $A \to B$ and $x \notin FV(M)$ then

$$\begin{aligned} \lambda x : A.\, Mx &\quad \to_\eta \quad M \\ M &\quad \to_{\bar\eta} \quad \lambda x : A.\, Mx \end{aligned}$$

long normal form $=$ $\beta\bar\eta$-normal form

$$\begin{aligned} \lambda f : a{\to}b.\, f &\quad : \quad (a \to b) \to a \to b \\ \lambda f : a{\to}b.\, \lambda x : a.\, fx &\quad : \quad (a \to b) \to a \to b \end{aligned}$$

theorem

every inhabited type has an inhabitant in *long* normal form

consistency

definition
a logic is called *inconsistent* if $\vdash A$ for *all* formulas A

theorem
minimal propositional logic is consistent

proof: analyze possibilities for β-normal forms M with $\vdash M : a$

β-normal form:

$$\lambda x : A.\, M'$$
$$x M_1 \dots M_k$$

both impossible:

a is not a function type
no variables in empty context

by Curry-Howard: $\nvdash a$ $\square$

recap

1. Curry-Howard
2. $\lambda\rightarrow$
3. minimal logic
4. styles of logic
   - Hilbert system
   - sequent calculus
   - natural deduction
     - Gentzen-style
     - Jaśkowsky/Fitch-style
5. detour elimination
6. consistency