Keys to two intimacies: Mathesis and Mysticism

Valedictory lecture by Henk Barendregt

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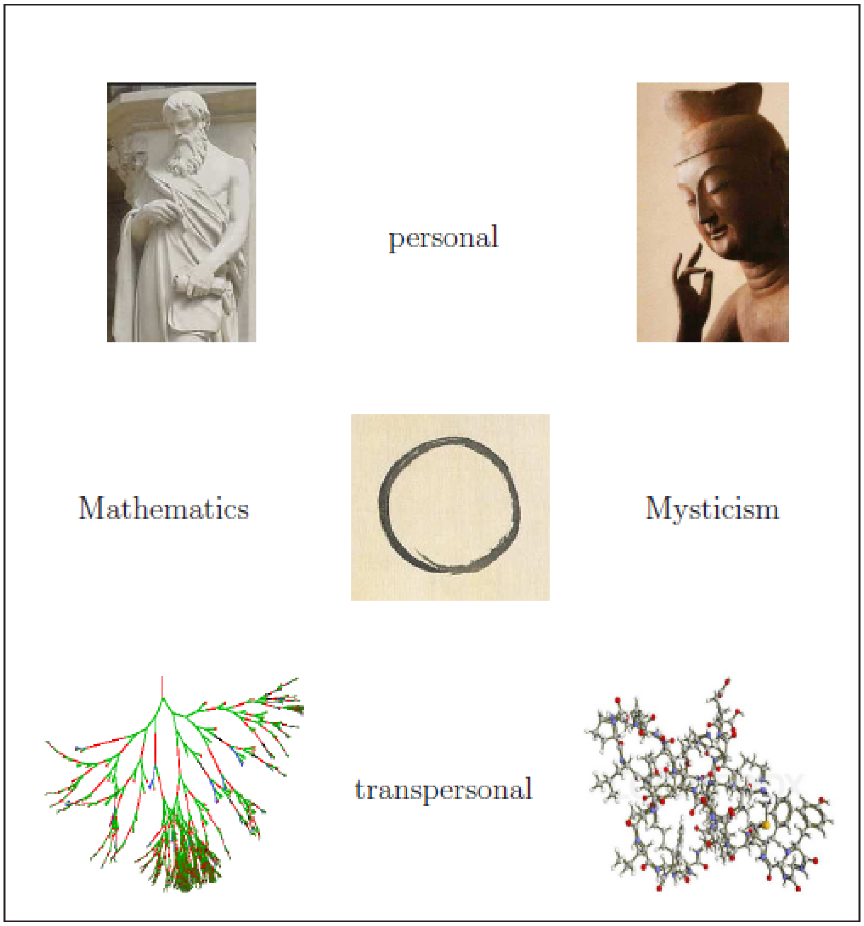
Lecture delivered at the farewell as Professor of Foundations of mathematics and computer science at the Faculty of Science, Mathematics and Computer Science of the Radboud University on Thursday, October 1st, 2015

by Henk Barendregt



From left to right and from top to bottom:
Euclid, Buddha, representation of emptiness,
formal proof that there are infinitely many primes, β-endorfine.

Mr Rector, Ladies and Gentlemen,
Esteemed Audience,

As *partir c'est mourir un peu*, we played Igor Stravinsky's Introitus In Memoriam T. S. Eliot.

The subjects mathematics and mysticism come from my main scientific interests: mathematics and meditation phenomena. We will discuss two types of keys for these topics: the personal, experienced from the inside, and transpersonal, described as an objective natural process. The two types of keys exist for both subjects.

In the novel Dr. Faustus of Thomas Mann two friends speak about human emotions and experiencing the divine in music. "You should love it," said the first. "Do you hold love as being the strongest emotion? " asked the other. "Do you know something stronger? " asked the former. "Yes, interest."

This passage shows that love and interest compete for being considered as the strongest emotion. This is sometimes described by romantics as the opposition between heart and intelligence or also between body and mind. But for most emotions a coordinated cooperation between body and mind is important.

The purest form of intelligence focuses on mathematics. The highest form of love is seen as the mystic unity with God. But to explain the mystical experience the hypothesis of the existence of the Supreme Being is not necessary. One can also understand it, as in Buddhist psychology, as a state of mind with a high form of concentration.

First there are the private keys to the two subjects. The experience of the mathematical truth through the mental activity of proving. This requires curiosity and study. The mystical experience is achieved through the practice of meditation, using the similar tools of motivation and commitment. For both experiencing mathematics and mysticism one only can create the right conditions, the rest is ---one could say---divine grace. In this way one describes it in many traditions of mysticism. But this parlance is also in vogue among mathematicians. Thus spoke Polish logician Andrzej Mostowski with admiration about his American colleague Robert Solovay: "He must have a direct phone line to God." So far the personal keys to mentioned subjects.

Then there are the transpersonal keys, stealing from both subjects as it were the soul. But I believe that the topics do gain depth through the combination of the personal and transpersonal aspects.

1. Two keys to mathematics

1.1 Personal

In his novel *The Man Without Qualities*, Robert Musil wrote the following about the main character, a mathematician who in his profession contemplates:

The precision, strength and security of this thinking that in life is unprecedented, he fulfilled almost

with melancholy[1].
We will give a simple example of this mathematical thinking.

1.1.1 Definition. A positive integer is called a *prime* if p>1 and p has only 1 and itself as divisor.

For example, of the numbers below ten 2, 3, 5 and 7 are prime; not prime are 1 (by definition), 4, 6, 8 and 9.

We do not want that 1 is prime for aesthetic reasons, which follows from the following fact, which we state without proof.

1.1.2. Proposition. *Each positive integer can be written as the product of a unique sequence (apart from the order, and possibly with duplicates or empty; 1 can be considered as the product of the empty sequence) of primes (decomposition into prime factors). Thus each number has been primedivisor, i.e. a divisor that is prime.*

For example 12=2x2x3. If one would considered 1 as a prime, then also 12=1x2x2x3, so that the decomposition into prime factors is no longer unique.

The ancient Greeks knew the primes. They could not only enumerate them, but also ask and answer questions about these. Consider the sequence of primes:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, ...

does this ever end or go on indefinitely?

1.1.3. Theorem (Euclid). *There are infinitely many primes*[2].
Proof. Consider a finite list of primes $p_1, \dots, p_n$. Define $P = p_1 p_2 \cdots p_n$. Laat q be a prime factor of P+1. Now p_1 divides P, so doesn't divide P+1. But q divides by definition P+1 and hence $q \neq p_1$. Similarly $q \neq p_2, \dots, p_n$. Hence q is a prime number that doesn't appear

1 *Die Genauigkeit, Kraft und Sicherkeit dieses Denkens, die nirgends im Leben ihresgleichen hat, erfühlte ihm Fast mit Schwermut*. R. Musil: *Der Mann ohne Eigenschaften*.
2 The result of Euclid on the infinity of the collection of primes immediately leads to another questions. A twin-prime is a pair of prime numbers with 2 as difference. Thus, (3,5) and (5,7) are prime-twins, but the pair (7.9) is not. An unanswered question is whether there exist infinitely many prime-twins. This simple question gives rise to complex mathematics. If one requires of two primes that they may differ up to 600, then there are infinitely many such couples, J. Maynard [2013] (arxiv.org/abs/1311.4600), where the number 600 an improvement of 70 million, previously found by Y. Zhang. In the meantime, the result appears to be improved in a joint project Polymath8 (arxiv.org/abs/1409.8361) to 246 as maximum difference. This way has not yet arrived at the difference 2 for the prime-twins and moreover the used methods can probably not reach it.

in the original list. Therefore every finite list of primes can be extended with a new one. We conclude that there are infinitely many primes. QED

A small variation of this proof shows, that for each number n a prime number can be found that is at most n!+1. In the for mathematics very rich 19-th century it even has been proven that the first prime number greater than n is at most 2n.
We can't control when mathematical insight arrives. The French mathematician Poincaré wrote in [1908] that he gave up to find a solution to a particular problem, after spending a long time on it. He was drafted for military service. After his military service he stepped on a good day in a tram. Placing his foot he suddenly saw the solution of his problem, which came to him via direct intuition. This still had to be verified rationally; it turned out to be right. Poincaré conjectured that his intuition came from subconscious. Later his colleague and countryman Hadamard [1945] stated that this subconscious intuition probably utilized parallel processing.

Although mathematics is considered to be the most precise of all sciences, the final verdict rests on a judgement based entirely on 'inner vision'. Despite the reasoning and calculations like in the proof above, it still needs the inner views to see the correctness of the reasoning, and the applicability of the calculation. This was true at least until the 20-th century. It now starts to change slowly.

1.2 Transpersonal
For the transpersonal keys to mathematics two ideas of the Greek philosopher Aristotle are of interest. His first contribution was the following description of the axiomatic method in mathematics. On the one hand there are mathematical objects (also known as concepts as these consist mainly in the mind), for example, numbers such as 2, ¾ and Ö5, or geometric figures such as triangles, ellipses and pyramids. On the other hand, there are qualities of those objects. It is worthwhile to find out what are the valid qualities, the so called theorems, that hold for an object or class of objects.

How does one arrive to mathematical objects and theorems about these? Objects are obtained from previously found objects by means of a definition. Theorems on finds from previously obtained ones by a proof. In order not to get into an infinite regression, one has to assume primitive concepts, that do not need a definition. Similarly, there are axioms, theorems that do not need not to be proven. On the basis of the primitive notions and axioms using definitions and and proofs one can develop a mathematical theory. The axiomatic method was considered in ancient Greece as follows: primitive concepts are so clear that they don't need a definition; similarly the axioms are so true that no proof is needed for them.

More than 2000 years later this view was improved by Hilbert, who gave an elegant interpretation of the axiomatic method. He considered the axioms as an implicit definition of the primitive concepts. It did not care what the essence of a point or a line, as long as there is exactly one line through two distinct points, one of the axioms of the Euclidean geometry, and also the axioms hold.

Proofs are based on intuitive reasoning. A second important contribution of Aristotle for the foundations of mathematics was his project to make a map of all possible ways of reasoning. This enterprise was only completed about 2300 years later by a proposal of Frege in [1879], of which Gödel showed in 1930 that Frege was right indeed. From that moment on proofs and hence the consequences of an axiomatic system were completely determined: a proof is a row of statements, which are guaranteed by the axioms or resulting from previous statements on the basis of the rules of logic.

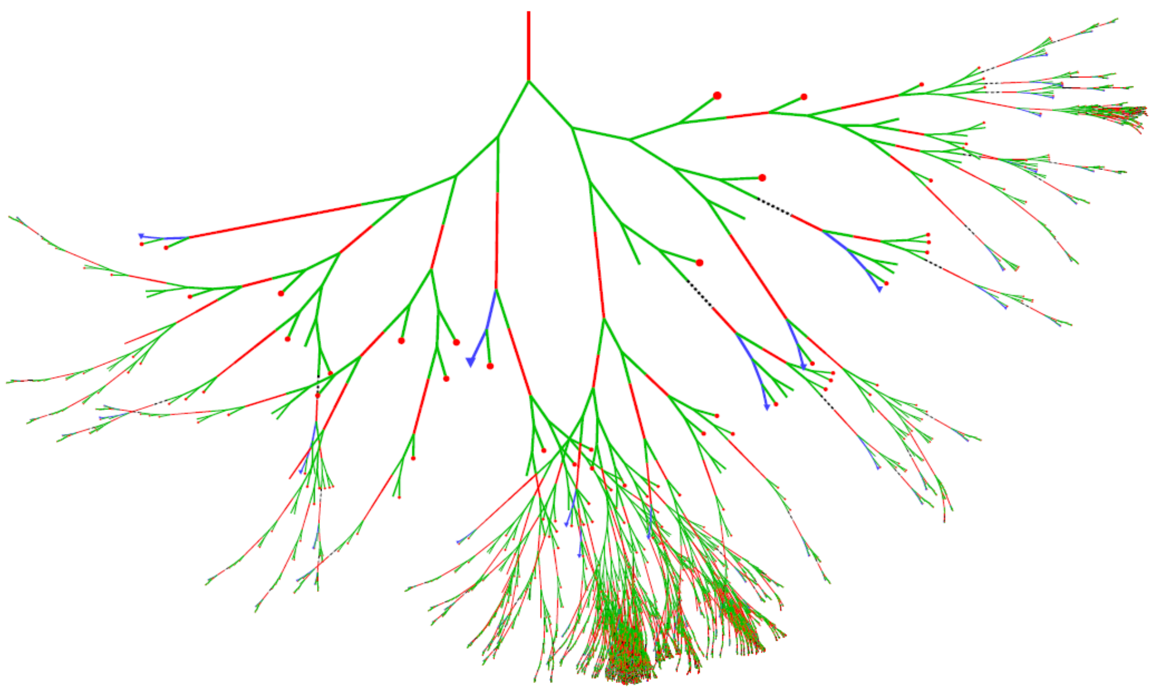


Fig. 1. Formal proof that there exist infinitely many primes. Formalisation: Freek Wiedijk; image: Joerg Endrullis. It is a tree with approximately 71 000 nodes (forkings and end-points). This seems too much: the intuitive evidence is short and the formal is long. We should be recalled that for the realization of an easy to perform operation, such as touching the tip of our nose with the index finger, many neurons and synapses will be used.

In this way, it is possible to write down proofs in a complete formal way and it is easy to determine objectively its validity: step by step. Then Frege [1893, 1903] began to formalize a (modest) part of arithmetic. Unfortunately, the logician Bertrand Russell found that a contradiction could be derived from the axioms used by Frege. This made the latter propose to withdraw his work but his publisher wanted to publish it anyway. After that Russell collaborated with the mathematician Whitehead to formalize a portion of mathematics (including arithmetic) within an axiom system considered to be safe. Thus resulted from the hand of these authors the first part of their monumental Principia Mathematica, Whitehead and Russell [1910]. This work, however, had the disadvantage that it is virtually unreadable; moreover Principia Mathematica was also formulated less precisely than the work of Frege. This being so, who verifies than the formal proofs of Principia Mathematica?

This question was answered by the Dutch mathematician NG the Bruijn [1970], who showed how a computer could be efficiently verify a formalization in a certain formalism. The required program is thereby very small, essentially a good description of the rules of the logic, which De Bruijn had displayed in a more convenient way than Frege. However, most mathematicians stayed away from formalizing: it was difficult, incomprehensible to humans, and the verified theorems were not advanced. In the beginning of the 90s of the last century it was still laborious to prove formally the follwing statements:

* 17 is a prime;
* (x+1)(x-1)=x2-1 holds for all real numbers;
* $(e^{x}+e^{-x})/2$ is a continuously differentiable function in the real variable x.

For a greater depth in formal mathematics new ideas were needed. These came from unexpected sources: (i) computer algebra systems; (ii) the method of proof of the incompleteness theorem of Gödel

As to (i). Computer Algebra Systems powerful, large, and therefore not always reliable. By using them in a skeptic way, similar to the role of intuition which is later to be verified, entirely reliable results can be obtained. This was presented in an article with Arjeh Cohen [2001]. This technique eventually led to the certification of primes of hundreds of digits. [First by Caprotti and Oostdijk [2001] for numbers with up to 100 digits, generating formal proof of their primality on the basis of the little theorem Fermat (Pocklington criterion). Thereafter, the method was extended to numbers with 300 digits on the basis of a decision method that uses elliptic curves, Thery et al. [2007].

As to (ii). As one of the few who had read and understood Principia Mathematica, Gödel [1931] has shown that there are statements, what can be neither proved nor disproved (unless used axioms lead to inconsistencies, such as the one used by Frege). This so-called incompleteness theorem seems to plea against formalization. The meaning is, however, that the axiomatic method has its limitation, regardless of whether proofs are formal or intuitive. The situation is that the axiomatic method is very strong, despite the incompleteness theorem. Moreover, we have nothing better: there is no algorithm that determine whether a statement is valid or not. This was proved by Turing [1937] with as spin off the design of the programmable computer, which most of you carry around in the form of a smartphone with apps. Gödel demonstrated this incompleteness result by encoding mathematical concepts and properties as numbers, so that the arithmetic can make indirect statements about themselves.

The application of the laws of logic is then a calculable operation on these numbers, which is representable within the axiomatic system for arithmetic. This so-called `reflection' was utilized in an article with Erik Barendses [2002] to generate formal proof on the basis of the syntactical properties of the statements. In this paper outlines are given how formal correctness proofs can be generated for arbitrary multiplication of polynomials. In Cruz-Filipe [2003] reflection is used to to generate evidence for continuity, differentiation and the calculation of the derivative of well-known real functions.

Formal proofs can be seen as keys, which open boxes containing fully developed mathematical understanding; the latter can be utilized for the construction of new more complex keys. There are also `dynamic keys', which may unfold during use to the correct shape. These correspond to proofs using calculations. The HOL system <en.wikipedia.org/wiki/HOL_ (proof_assistant)> for formal mathematics employs dynamic keys must first be unfolded, before verification is possible.

In other systems, such as Coq <coq.inria.fr>, which does not always work, because this one also calculation model contains a programming language, which can be reasoning and advance proof that an expanded key are work will do. Is it an efficient dynamic proof key example, there are parameterized in Coq proof and can be shown to all natural numbers expression is evidence of a certain quality without having to be expanded. The already de Bruijn said the lambda calculus of a new typing equipped to represent such evidence. After expansion with operators for higher order primitive recursion, Gödel introduced in [1958], are efficient dynamic proof keys become possible.

One of the first successful combinations of a logical system with external computing device is in an article by Gödel [1958], where an operator for higher-order primitive recursion is introduced.

Reflection together with efficient expandable proof keys are in the bases used for the research in Nijmegen, obtaining formal proofs, including fo mathematical analysis.[3]

Georges Gonthier, one of the speakers at this morning's symposium, this method was used as an important tool for his impressive formalizations of the Four color theorem in combinatorics [2008] and the theorem of Feit-Thompson [2014] from group theory. By Hales et al. [2015] another *tour de force* has been established, both from the intellectual and organizational[4] point of view.

This consists of the formalization of Hales' proof of the Kepler conjecture about the most efficient space packing of spheres. The Foundations Department can be proud that Hales has spent a sabbatical in Nijmegen and that two former colleagues from the department

3 Constructive proofs of the fundamental theorem of algebra (Geuvers et al. [2002]) and analysis (Cruz-Filipe [2003]). In O'Connor [2008] certified proofs are given of efficient algorithms for infinitely precise calculations with real Numbers.

4 Because many people have helped the formalization, especially a group of students in Vietnam. To some of these a PhD position was offered at the University of Pittsburgh, where Hales works.

are co-author of the groundbreaking article. It can rightly be said that formalizing of proofs is possible for 'non-trivial' (an understatement) results. However, with much effort: according to an estimate of Freek Wiedijk the needed time to construct a formalization of a proof is on the average about ten times as long as writing it down informally.

1.3 A comparison

The first reaction on formalized proofs by many mathematicians was disapprovement. Because usually proofs are developed in their consciousness, one considered formal proofs, which are usually too large to be overlooked entirely, as a treason against the spirit of mathematics. Such an attitude undoubtedly also must have existed in the early days of cellular biology. While a practitioner of 'natural history' first romantically went to the fields to catch plants and animals for studying, later biological research was carried out by using microscopes.

Because of this reluctance to formalize, the automated verification of proofs in mathematics was preceded by that of correctness proofs of first hardware and later software. For this interactive systems were built, the so-called *proof assistants*, which are helpful in construction of formal proofs within IT[5]. These applications have been instrumental for the rise of formal mathematics.

My goal during the occupation of the chair of Foundations was to make more common the act of providing formal proofs in mathematics. It was my expectation that by steadily working on the formalization of the Master's topics in mathematics at some point enough, say> 50 percent, would be verified, after which there would be interest in fellow mathematicians. But things went differently. Despite the fact that only a very small percentage of the Master's curriculum has been formalized, nowadays even Fields Medal winners (V. Voevodsky <arxiv.org/abs/1401.0053>) use mechanically verified proofs, because also for them the arguments used may be so complex that mechanical verification is needed. What can be used as the most appropriate logic is not yet clear. Mentioned facts place the logic and foundations in a central place among the research and applications of mathematics.

The precision of the formal proofs explains in my view why mathematics is so very reliable. That informal proofs provide a very reasonable approximation is a particular feature of the human thinking. I expect that a collaboration between human intuition and machine verification will bring mathematics and computer science to a new level of precision, without loosing their intellectual excitement.

A major challenge is to make formalizing more user-friendly. Right now it is still very time-consuming. Offering a manuscript for publication at present is occasionally combined

5 The company Intel needed to reserve US$ 400M for claims, because they had brought a Pentium chip market in 1994 which was a mistake. That gave a boost to machine authentication to prove that a design meets a particular specification.

with a formalization of the proofs. In such cases a human referee will still be necessary, in order to determine whether the statement is formalized correctly and whether the results are interesting, for example because they can be related to other work.

2. Two keys to mysticism

Mysticism is often confused with mystification. This is because mystical experiences are difficult to express verbally. Mysticism is therefore often seen as irrational. Also, mystical experiences are considered to be *anti*-rational, because rational thinking is held to be inadequate to induce mystical experiences; moreover, rational thinking is in fact correctly considered as a hindrance to mystical experiences. The Dutch philosopher Frits Staal (1930-2012) has a more down to earth take on mystical experiences. In his book *Exploring mysticism* (1975) he asserts that mysticism forms part of basically every culture. However, every culture has a different way of seeing and integrating mystical experiences. In addition, Staal holds that the mystical experience is neither rational, nor irrational: it refers to the experiential and may be studied as such.

2.1 Personal domain

Induction

We set out to describe one of the possible personal keys to mystical experience: Buddhist insight meditation. However, first we need a few words on the phenomenon of 'consciousness'. Consciousness has two distinct features: 1. It is directed towards something, which is called the *object o*f consciousness. The object is based on data coming to us through the physical senses. Like for instance, through our ears, eyes, or through the mind itself, like in the case of memories. 2. Then there is the mental disposition towards the object, also known as the *mental state.* The mental state determines the 'colouring' of our consciousness. Then the mental state, accompanied by its corresponding colour, determines the processing of the object and what direction the following actions will take. In the meditation practice, a friendly discipline limits the stream of incoming objects as much as possible: in a quiet environment, one sits still with eyes closed. In order to quiet down the input coming from the content of our mind, one concentrates on the bodily movement caused by the neutral act of breathing. Nevertheless, sooner or later, one may get distracted and engage in a train of thought. Once this is noticed, the attention is kindly directed back to the breathing. This practice, if systematically repeated (and if our lifestyle is adjusted accordingly), can give rise to a particular, yet general calm. This calm allows for a better governance of the direction of our attention. However, through the emergence of particular mental states, several hindrances may occur, including desire, aversion, sleepiness and restlessness. Each of these hindrances is accompanied by a tendency to abandon the practice schedule. If one nevertheless continues to practice (observing "Ah, there is pain.", or "Ah, there is restlessness."), a mental state may arise in which none of the hindrances is present. Such a mental state forms the beginning, the initiation of certain mystical experiences.

The phenomena

The phenomena that typify a mystical experience are known to a large variety of traditions and are often described in terms of glory and rejoice. These traditions range from monistic religions such as Christianity, poly- and a-theistic religions like Hinduism or Buddhism, to shamanistic cultures. We here confine ourselves to the sometimes rather aloof descriptions of mystical states by classical Buddhism.

The so called *jhānas (absorptions)* form part of what may be called the Buddhist mystic experience. During the first *jhāna* the state of consciousness of the practicioner contains the following substates: continuous attention, rapture, joy and concentration. Maintaining continuous attention requires an amount of energy and can be abandoned after a while, which will lead to the entering of the second *jhāna.* Rapture or ecstasy adversely give rise to agitation. Dissociation of it may lead to the entering of the third *jhāna*. The third *jhāna* consists of merely joy and concentration. Once the joy is abandoned as well, sheer equanimous concentration remains. These different forms of *jhāna's* are consistent with 'mystic experience' known from other cultures, such as the descriptions of Theresia of Avilla versus that of Meister Eckhart.

In any case, the mystical experience in all its variety is held to be soothing and beneficial for body and mind. It is also known that reducing stress may have many beneficial effects, including enhancing the functionality of the immune system. Subsequently, research on the effects of meditation has seen an exponential growth internationally.

2.2 Transpersonal

There are indications that some of the phenomena that occur during a mystical state, can be explained by an increased concentration of neuromodulators in the brain, see e.g. Veening & Barendregt [2015]. Especially β-endorfine has an overall calming and analgesic effect on mind and body. This is related to the fact that β-endorfine binds to the same neuroreceptors as morphine, which is the active component of opium.

It seems that what can be experienced through personal endeavours in meditation, can be explained in terms of universal neurophysiological mechanisms of a group of molecules. The latter can thus be seen as transpersonal keys to mysticism.

2.3. Deepening

But wait a minute. Karl Marx gives a pause for thought here: “Religion is the opium of the masses”. According to Marx, religion - including mystical states - can be used by a ruling despot to delude and quiet down the masses. The above-mentioned explanation of the reduction of stress by endogenous opiates would even provide Marx’s claim with neurophysiological support. However there is more to say.

In the tradition of insight meditation, the mystical experience is seen as a distraction as well: the goal is not temporarily ecstasy and relief of uneasiness by suppression; rather the practice objective is to cultivate and constitute sustainable equanimity. The important Spanish mystic St. John of the Cross, whose path has been carefully compared to the path of the insight meditation by Meadow & Culligan [1987], writes that *"the way ... does not consist of fun, experiences and spiritual feelings.* " Temporary ecstatic experiences can become distractive addictions. They prevent the occurrence of wholesome sustainable changes in our personality.

Conditioning (personal)

What kind of changes are we to think of? All living organisms are ‘conditioned’. Even single-celled organisms are conditioned clearly by avoiding poison and approaching nutrition. This the adaptive mechanism of desire and aversion on a very primitive level. As another example, many insects are conditioned as follows: they navigate by maintaining a fixed angle to a bright light. When this source of light and orientation is at an infinitely far distance, such as the sun or the moon, the insect will fly in a straight line and navigate efficiently. This adaptive conditioning, however, became life threatening from the moment that Homo Sapiens started to make fires. Light sources weren’t necessarily on an infinite far distance anymore. While maintaining a fixed angle towards it, insects will fly in a decreasing spiral around a light source, will eventually end up inside the light source and burn.
Likewise, we, as human being are conditioned, and not necessarily in an adaptive manner. We all know someone who tends to do things that he or dhe knows one shouldn’t do it, but can’t resist doing it anyway. And being honest, we have to admit that we even sometimes do that ourselves. In other words, we are not free.

Deconditioning

Certain species of insects have evolved and learned not to fly into the flame. This adjustment happens on the time scale of evolution, over the course of many generations. In contrast, Homo sapiens, has a trump card using which one can decondition in the span of one lifetime. This happens in three phases.
1. By means of strong concentration that can be obtained by said meditation exercises. With strong concentration we raise the resolution of observing our internal mental and bodily

phenomena. At a certain point, we can experience that the stream of consciousness is not progressing in a continuous manner, but rather, that it is pulsating, like an impersonal mechanism. Our intentions are irellevant, also these are subjected to the pattern of the pulsing machine. They are not as independent as we might hold them to be. There is no fixed 'self' that governs this pattern. The experience of the absence of 'self' is called 'emptiness'. The self does exist, but merely as a construct made up of coordinating modules. It requires quite some energy to sustain the construct and coordination of the self. When the required energy to sustain the construct is not available, like during periods of stress or illness, it may happen that the construct temporary collapses and emptiness is experienced.

2. One clings to the illusion of the fixed self in order to avoid the experience of emptiness. Nevertheless, once emptiness is experienced and perceived, its image is so deeply encrypted in us, that ignoring it is no longer an option. However, our inner resistance to the experience of emptiness is persistent and gives rise to suffering. After St. John of the Cross, this suffering is referred to as "the dark night of the soul". By means of systematic concentration and continuous mindfulness, one can learn to let go of the resistance to the experience of emptiness. At first this is temporarily. By letting go one experiences the so-called 'phase of equanimity.

3. Eventually we may see what lies at the root of our resistance and the consequent suffering, namely a *wrong view*. In the wrong view we see ourselves as the centre of the universe and as a fixed self in optimal control of our surroundings. In order to let go of the resistance to the right view, one needs curiosity and mild surrender that consequently will cause our consciousness and behavior to be more flexible.

It is almost a paradox that our conditioning decreases by acknowledging that we are fully conditioned! This can be explained as follows. Because we resist to the idea of being fully conditioned, we make all kinds of wrought manoeuvres driven by desire and aversion, to the effect of covering up the state of not being in control. Abandoning the habit of compulsory generating this waste of energy, gives rise to calmness and new ways to organize our lives.

Towards a scientific explanation (transpersonal)

In the fields of the cognitive neurosciences it is well known that all brain activity is determined by former neurophysiological activity and currently experienced input. Besides, there are indications that perception doesn't occur continuously, but in a pulsating manner. It is less known though, that these assertions can be experienced phenomenologically and that such an experience is difficult to come to term with

This is because it doesn't correspond with the view of the self that we generally hold. The unmasking of the wrong view lies beneath the surface of any state of consciousness and is avoided by entertaining and maintaining a view of the self as a fixed and continuous entity. To this end, old self-affirming habits are employed frequently. A simple explanatory hypothesis is that the performance of these habits and tendencies yield a dose of endogenous opiates that keep the void (emptiness)out of sight. These habits are addictive and that is the reason why it can be so difficult to let go of old habits. Many tendencies and habits are characterised by "replacing fear for no-thing, by fear for some-thing", as the psychologist R. May [1950] neatly phrases it. This goes to the extend of initiating wars and severely polluting the environment; but also engaging in irrational anxieties and thoughts are part of it. The above-mentioned addiction also explains why at this precarious moment in the history of mankind, politicians persistently remain incapable of taking the necessary measures, that obviously will serve the benefit of all.

We will now leave the cause of suffering behind and proceed with the mentioned solution. Is this solution plausible? Several authors, including Zylberberg, Dehaene et al (2011), propose the brain as a hybride Turing machine: pulsating according to prefixed patterns, while a neural net determines the intermediate steps. This view of the functioning of the brain is consistent with the meditative experience of observing consciousness as a deterministic pulsating stream of input. Barendregt & Raffone (2013) further extend this proposed model to the possibility of observing one's own mental content or consciousness. The latter is essential to mindfulness. With the development of mindfulness, an extra sense is cultivated that acts as a radar for our automatic pilot. The new developed sense allows for the possibility of adjusting the internalised mental programmes. This explanation suffices for the possibility of letting go of persisting habits, including the clinging to the wrong view of self.

The currently flourishing field of meditation research will hopefully be able to present results in the said direction before too long. Meditation experience can act as intuition acts in mathematics; as a source for defining hypotheses that can consequently be followed up and verified experimentally. However, before that may happen, humanity finds itself collectively in the state of the 'dark night of the soul'. But there is confidence that there will be light at the end of the tunnel. Let us now listen to a number of verses of the poem by the hand of st. John of the Cross: "Cantar del Alma", through which such confidence is expressed. The Italian composer Sergio Militello has intriguingly put this poem to music.

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Program Farewell Lecture

Pavane Mélancolique (2010)	**K. Schoonenbeek (1947)**
Lidia van der Vegt	Oboe
Appelona Klarenbeek	Flute
Heleen Venekamp	Harp

Introitus (1965)	**I. Strawinsky (1882-1971)**
Wouter van Haaften	Direction
Heleen Venekamp	Harp
Rik Helmes	Piano
Anne Wiersum	Altviool
Jacinta Molijn	Contrabas
Luís Cruz-Filipe	Tenor
Emil Hoefnagel	Tenor
Ruurd Lof	Tenor
Jan-Jetze Zijlstra	Tenor
Matteo Sammartino	Bass
Jacques van der Smissen	Bass
Bernd Souvigner	Bass
Jozef Steenbrink	Bass
Henk Barendregt	Timpani
Walter van Suijlekom	Timpani
Toine Schreurs	Tam-tam
Pieter Lamers	Tam-tam

Afscheidsrede Henk Barendregt

Cantar del Alma (2006)	**S. Militello (1968)**
Emil Hoefnagel	Tenor
Lidia van der Vegt	Oboe
Appelona Klarenbeek	Flute
Heleen Venekamp	Harp

Text *Introitus* and *Cantar del alma* next page.

Introitus (In Memoriam T.S. Eliot)

Requiem aeternam dona ei, Domine.
Et lux perpetua luceat ei.
Te decet hymnus, Deus, in Sion,
Et tibi reddetur votum in Jerusalem
Exaudi orationem meam
Ad te omnis caro veniet.
Requiem aeternam dona ei, Domine.
Et lux perpetua luceat ei.

Cantar del alma que se huelga de conoscer a Dios por fe

Juan de la Cruz (1542-1591)

Ritornello:
Que bien sé yo la fonte que mana y corre:
aunque es de noche.

1. Aquella eterna fonte está escondida,
que bien sé yo dó tiene su manida,
aunque es de noche.

2. Su origen no lo sé, pues no le tiene,
mas sé que todo origen de ella viene,
aunque es de noche.

Ritornello

3. Sé que no puede ser cosa tan bella,
y que cielos y tierra beben de ella,
aunque es de noche.

4. El corriente que nace de esta fuente
bien sé que es tan capaz y omnipotente,
aunque es de noche.

Ritornello

5. Aquí se está, llarnando a las criaturas,
y de esta agua se hartan,
aunque a oscuras, porque es de noche.

6. Bien sé que suelo en ella no se halla,
y que ninguno puede vadealla,
aunque es de noche.