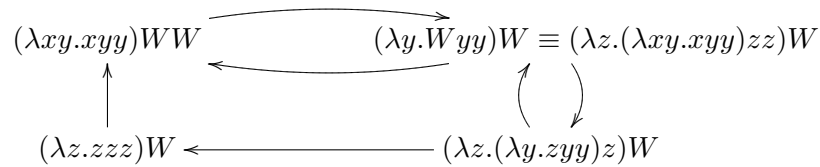


## Solutions Lambda Calculus (week 5, 11.12.2013)

### Exercise 1

$\mathcal{G}(WWW)$  is de volgende graph.



### Exercise 2

1. Zij  $F_* \equiv \lambda mnfx. m(nf)x$  en  $\mathbf{c}_n \equiv \lambda fx. f^n x$ .

$$F_* \mathbf{c}_2 \mathbf{c}_3 \equiv (\lambda mnfx. m(nf)x) \mathbf{c}_2 \mathbf{c}_3 = \lambda fx. \mathbf{c}_2(\mathbf{c}_3 f)x = \lambda fx. (\mathbf{c}_3 f)^2 x = \lambda fx. \mathbf{c}_3 f(\mathbf{c}_3 f)x = \lambda fx. \mathbf{c}_3 f(f^3 x) = \lambda fx. f^3(f^3 x) = \lambda fx. f^6 x = \mathbf{c}_6.$$

2.  $\mathbf{c}_2 \mathbf{c}_3 = (\lambda fx. f^2 x) \mathbf{c}_3 = \lambda x. \mathbf{c}_3^2 x$ ;

$$\mathbf{c}_3^2 x = \mathbf{c}_3(\mathbf{c}_3 x) = (\lambda fy. f^3 y)(\mathbf{c}_3 x) = \lambda y. (\mathbf{c}_3 x)^3 y;$$

$$(\mathbf{c}_3 x)^3 y = \mathbf{c}_3 x(\mathbf{c}_3 x(\mathbf{c}_3 x y)) = \mathbf{c}_3 x(\mathbf{c}_3 x(x^3 y)) = \mathbf{c}_3 x(x^3(x^3 y)) = x^9 y, \text{ omdat } \mathbf{c}_3 x \square = x^3 \square.$$

$$\text{Dit alles invullen geeft } \mathbf{c}_2 \mathbf{c}_3 = \lambda x. \lambda y. x^9 y \equiv \lambda fx. f^9 x = \mathbf{c}_9.$$

3.  $\mathbf{c}_3 \mathbf{c}_2 = (\lambda fx. f^3 x) \mathbf{c}_2 = \lambda x. \mathbf{c}_2(\mathbf{c}_2(\mathbf{c}_2 x))$   
 $= \lambda x. \mathbf{c}_2(\lambda z. (\mathbf{c}_2 x)^2 z) = \lambda x. \mathbf{c}_2(\lambda z. \mathbf{c}_2 x(\mathbf{c}_2 x z)) = \lambda x. \mathbf{c}_2(\lambda z. x^2(x^2 z))$   
 $= \lambda x. \mathbf{c}_2(\lambda z. x^4 z) = \lambda x. (\lambda w. (\lambda z. x^4 z)^2 w) = \lambda x w. x^4(x^4 w)$   
 $= \lambda x w. x^8 w = \mathbf{c}_8.$

[Hoe bewijst je  $F_* \mathbf{c}_n \mathbf{c}_m = \mathbf{c}_{n.m}$  en  $\mathbf{c}_n \mathbf{c}_m = \mathbf{c}_{m^n}$  voor alle  $n, m \in \mathbb{N}$ ?

### Exercise 3

$$Mx = xMx \iff M = \lambda a. aMa \iff M = (\lambda ba. aba)M$$

Zij dus  $G = (\lambda ba. aba)$ , dan  $M = GM$ . Ofwel  $M$  is een fixed point van  $G$ .  
 Definieer  $M = WW$  met  $W = \lambda c. G(cc)$ .

$$M = (\lambda cba. aba(cc))(\lambda cba. aba(cc))$$

Nu geldt  $Mx \rightarrow (WW)x \rightarrow ((\lambda c. G(cc)W)x \rightarrow G(WW)x \rightarrow GMx \rightarrow (\lambda ba. aba)Mx \rightarrow (\lambda a. aMa)x \rightarrow xMx$ .