Exercises Lambda Calculus (week 5, 11.12.2013)

Exercise 1

In order to know lambda terms well, we introduce reduction. This gives a direction to equality. There is a one-step reduction $\to$, more-step reduction $\twoheadrightarrow$ (0, 1 or more steps).

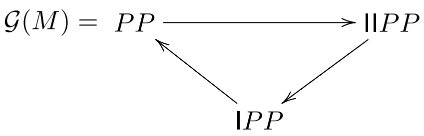
$$\boxed{\begin{array}{rcl}(\lambda x.M)N \to M[x:=N] & & \\ N \to N & \Rightarrow & M \twoheadrightarrow N \\ M \twoheadrightarrow M & & \\ M \twoheadrightarrow N \;\&\; N \twoheadrightarrow L & \Rightarrow & M \twoheadrightarrow L \\ M \twoheadrightarrow N & \Rightarrow & MZ \twoheadrightarrow NZ \\ M \twoheadrightarrow N & \Rightarrow & ZM \twoheadrightarrow NZ \\ M \twoheadrightarrow N & \Rightarrow & \lambda x.M \twoheadrightarrow \lambda x.N \end{array}}$$

Examples (i)
$$\begin{array}{rcl}\mathsf{I}x & \to & x; \\ \mathsf{II}x & \to & \mathsf{I}x \\ & \to & x; \\ \mathsf{II}x & \twoheadrightarrow & x.\end{array}$$

(ii) Given $M \in \Lambda$, write $\mathcal{G}(M)$, the graph of M, for

$$\{N \mid M \twoheadrightarrow N\}$$

with the relation $\to$ displayed on its elements. For example let $P \equiv \lambda x.\mathsf{II}xx$ and $M \equiv PP$. Then



Now the exercise. Let $W \equiv \lambda xy.xyy$. Draw $\mathcal{G}(WWW)$.
[Hint. This graph consists of exactly four terms.]

Exercise 2

Let $F_* \equiv \lambda mnfx.m(nf)x$ and $\mathbf{c}_n \equiv \lambda fx.f^n x$. Compute $F_*\, \mathbf{c}_2\, \mathbf{c}_3,\ \mathbf{c}_2\mathbf{c}_3,\ \mathbf{c}_3\mathbf{c}_2$.

Exercise 3

Write down precisely a lambda term M such that

$$Mx = xMx.$$

Can you make it satisfy $Mx \twoheadrightarrow xMx$?