# Exercises Lambda Calculus (week 6, 18.12.2013)

## Exercise 1

1. Given is the data type $\texttt{Nat}$ with

   $\texttt{z} : \texttt{Nat}$, $\texttt{s} : \texttt{Nat} \to \texttt{Nat}$.

   Write down the codes (following Böhm, Guerrini, Piperno) of

   $$2 = s(sz),\ 3 = s(s(sz)).$$

2. Predecessor on $\texttt{Nat}$ can be defined recursively:

   $$\begin{aligned} p(0) &= 0 \\ p(n+1) &= n. \end{aligned}$$

   Using the theory you learned, constrct a term $P$ of the form $\langle\langle B_1, B_2\rangle\rangle$, to act on codes of $\texttt{Nat}$, such that

   $$\begin{aligned} P(\ulcorner z \urcorner) &= \ulcorner 0 \urcorner \\ P(\ulcorner sn \urcorner) &= \ulcorner n \urcorner. \end{aligned}$$

3. Verify $P\ulcorner 3 \urcorner = \ulcorner 2 \urcorner$.

## Exercise 2

1. Given is $\texttt{Tree}$, the data type with

   $$\texttt{l} : \texttt{Tree}, \texttt{p} : \texttt{Tree}^2 \to \texttt{Tree}.$$

   Write down the codes (following BGP) of

   $$\texttt{t}_1 = \texttt{p}(\texttt{pll})\texttt{l};\ \texttt{t}_2 = \texttt{pl}(\texttt{pll}).$$

2. Write down a $\lambda$-term $F = \langle\langle D_1, D_2\rangle\rangle$ (to act on codes of $\texttt{Tree}$) such that

   $$\begin{aligned} F\ulcorner l \urcorner &= l \\ F\ulcorner pts \urcorner &= \ulcorner pt(pts) \urcorner. \end{aligned}$$

3. Verify for the $F$ you found that indeed $F\ulcorner pl(pll) \urcorner = \ulcorner pl(pl(pll)) \urcorner$.

## Exercise 3*

Let $E = \langle\langle \mathsf{K}, \mathsf{S}, \mathsf{C}\rangle\rangle$, with $\mathsf{C} = \lambda xyz.xzy$. Assume $E\ulcorner M \urcorner = M$. Show

$$E\ulcorner \lambda x.M \urcorner = \lambda x.M.$$

[Remember that $\ulcorner \lambda x.M \urcorner = \texttt{Abs}\ (\lambda x.\ulcorner M \urcorner)$, with $\texttt{Abs} = \lambda xe.eU_3^3xe$.]