

Exercises Lambda Calculus (week 7, 08.01.2014)

1. Define **true** $\triangleq \lambda xy.x(\equiv K)$ and **false** $\triangleq \lambda xy.y(\equiv KI)$.

(a) Given λ -terms P, Q , construct a λ -term $F_{P,Q}$ such that

$$\begin{aligned} F_{P,Q}\mathbf{true} &= P; \\ F_{P,Q}\mathbf{false} &= Q. \end{aligned}$$

(b) Construct a λ -term F_{neg} such that

$$F_{\text{neg}}\mathbf{true} = \mathbf{false} \ \& \ F_{\text{neg}}\mathbf{false} = \mathbf{true}.$$

(c) Construct a term F_{and} such that

$$\begin{aligned} F_{\text{and}}\mathbf{true} \ \mathbf{true} &= \mathbf{true}; \\ F_{\text{and}}\mathbf{true} \ \mathbf{false} &= \mathbf{false}; \\ F_{\text{and}}\mathbf{false} \ \mathbf{true} &= \mathbf{false}; \\ F_{\text{and}}\mathbf{false} \ \mathbf{false} &= \mathbf{false}. \end{aligned}$$

Every λ -term is either of the form

2. (a) Construct a λ -term H such that

$$\begin{aligned} G^\top x^\top &= \mathbf{true} \\ G^\top PQ^\top &= \mathbf{false} \\ G^\top \lambda x.P^\top &= \mathbf{false}. \end{aligned}$$

(b) Construct a λ -term V such that

$$\begin{aligned} V^\top x^\top &= \mathbf{true} \\ V^\top PQ^\top &= V^\top P^\top \\ V^\top \lambda x.P^\top &= \mathbf{false}. \end{aligned}$$

(c) Construct a λ -term H such that

$$\begin{aligned} H^\top x^\top &= \mathbf{true} \\ H^\top PQ^\top &= V^\top P^\top \\ H^\top \lambda x.P^\top &= H^\top P^\top. \end{aligned}$$

(d) Compute $H^\top S^\top$, $H^\top Y^\top$, where $Y \triangleq \lambda f.(\lambda x.f(xx))(\lambda x.f(xx))$.

(e) Note that $Y = \lambda f.f((\lambda x.f(xx))(\lambda x.f(xx)))$. Compute

$$H^\top \lambda f.f((\lambda x.f(xx))(\lambda x.f(xx)))^\top.$$

3.* (a) Show that every λ -term M is either of the form $M \equiv \lambda x_1 \dots x_n.yQ_1 \dots Q_m$ (with *head variable* y) or $M \equiv \lambda x_1 \dots x_n.(\lambda y.P)Q_0Q_1 \dots Q_m$ (with *head redex* $(\lambda y.P)Q$). In the first case M is a *head normal form*.

(b) Show that in the first respectively second case one has $H^\top M^\top = \mathbf{true}$ and $H^\top M^\top = \mathbf{false}$.