MRI master class Computer Mathematics

For the course notes see `http://www.cs.kun.nl/ herman/` under "Education", more specifically: `http://www.cs.kun.nl/ herman/SLIDES/lec1.ps.gz`

Exercises 1: Simple Type Theory

1. (a) Assign a type to $\lambda x\, y.yxx$

   (b) Assign a type to $\lambda x\, y.x(\lambda z.y)y$

2. Find inhabitants of the following types

   (a) $(\alpha{\to}\beta){\to}(\beta{\to}\gamma){\to}\alpha{\to}\gamma$

   (b) $((\alpha{\to}\beta{\to}\alpha){\to}\alpha){\to}\alpha$

3. Compute the principal type of the following terms

   (a) $\lambda x\, y\, z.xz(yz)$ (the combinator **S**)

   (b) $\lambda x\, y.x$ (the combinator **K**)

   (c) **SKK**

   (d) $\lambda x.x$

   (e) Compute the normal form of **SKK** and make an observation.

4. (a) Show that there are infinitely many closed terms (inhabitants) of the type $\alpha{\to}(\alpha{\to}\alpha){\to}\alpha$

   (b) Describe the derivations that the (infinitely many) terms under (4.a) correspond to.

   (c) Construct a derivation of $((\alpha{\to}\beta){\to}\gamma){\to}\beta{\to}\gamma$ and the associated typed λ-term.

5. Prove the three parts of the Lemma in the Strong Normalization proof (slide 24).

   (a) $[\![\sigma]\!] \subseteq \mathsf{SN}$

   (b) $x^\sigma \in [\![\sigma]\!]$ (Hint: Prove, by induction on σ, the following stronger result. If $\tau = \tau_1{\to}\ldots{\to}\tau_k{\to}\sigma$ and $x^\tau P_1 \ldots P_k : \sigma$ with $P_i \in [\![\tau_i]\!]$, then $x^\tau P_1 \ldots P_k \in [\![\sigma]\!]$.)

   (c) If $M \in [\![\sigma]\!]$, $N \in [\![\tau]\!]$, $M[N/x] \in \mathsf{SN}$, then $M[N/x] \in [\![\sigma]\!]$.