Exercises "Introduction to Type Theory" 1

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On simple type theory à la Church

1. (basic) Find a term of type $(\delta \to \delta \to \alpha) \to (\alpha \to \beta \to \gamma) \to (\delta \to \beta) \to \delta \to \gamma$
2. (basic) Find two terms of type $(\delta \to \delta \to \alpha) \to (\gamma \to \alpha) \to (\alpha \to \beta) \to \delta \to \gamma \to \beta$
3. (medium) Add types to the λ-abstractions and give the derivation in minimal logic that corresponds to
$$\lambda x.\lambda y.y(\lambda z.y\, x) : (\gamma \to \epsilon) \to ((\gamma \to \epsilon) \to \epsilon) \to \epsilon$$
4. (medium) Find a term of type $((\alpha \to \beta) \to \alpha) \to (\alpha \to \alpha \to \beta) \to \beta$

On simple type theory à la Curry

1. (medium) Give a full derivation of
$$\vdash \lambda x.\lambda y.y(\lambda z.y\, x) : (\gamma \to \epsilon) \to ((\gamma \to \epsilon) \to \epsilon) \to \epsilon$$
in Curry style $\lambda\!\to$
2. (medium) Determine the most general unifiers of
   (a) $(\alpha \to \beta) \to \gamma$ and $\alpha \to \beta \to \gamma$
   (b) $(\alpha \to \beta) \to \gamma$ and $\gamma \to \alpha \to \beta$
3. (medium) Compute the principal type of $\mathbf{S} := \lambda x.\lambda y.\lambda z.x\, z(y\, z)$.
4. (medium) Which of the following terms is typable? If it is, determine the *principal type*; if it isn't, show that the typing algorithm fails.
   (a) $\lambda z\, x.z(x(\lambda y.y\, x))$
   (b) $\lambda z\, x.z(x(\lambda y.y\, z))$
5. (advanced) Compute the principal type of $M := \lambda x.\lambda y.x(y(\lambda z.x\, z\, z))(y(\lambda z.x\, z\, z))$.
6. (advanced) Verify the details of the weak normalization proof (see the slides)
7. (advanced) Verify the details of the strong normalization proof (see the slides)

On polymorphic type theory (medium)

Recall: $\bot := \forall\alpha.\alpha$, $\top := \forall\alpha.\alpha\to\alpha$.

1. (basic) Verify that in Church $\lambda 2$: $\lambda x{:}\top.x\top x : \top\to\top$.
2. (medium) Verify that in Curry $\lambda 2$: $\lambda x.xx : \top\to\top$
3. (medium) Find a type in Curry $\lambda 2$ for $\lambda x.x\,x\,x$
4. (medium) Find a type in Curry $\lambda 2$ for $\lambda x.(x\,x)(x\,x)$

Recall $\sigma\times\tau := \forall\alpha.(\sigma\to\tau\to\alpha)\to\alpha$, $\sigma+\tau := \forall\alpha.(\sigma\to\alpha)\to(\tau\to\alpha)\to\alpha$ $\mathrm{Tree}_{A,B} := \forall\alpha.(B\to\alpha)\to(A\to\alpha\to\alpha\to\alpha)\to\alpha$

1. (medium) Define $\mathrm{inl} : \sigma \to \sigma + \tau$
2. (medium) Define the first projection: $\pi_1 : \sigma \times \tau \to \sigma$
3. (advanced) Define $\mathrm{join} : \mathrm{Tree}_{A,B} \to \mathrm{Tree}_{A,B} \to A \to \mathrm{Tree}_{A,B}$