Exercises "Introduction to Type Theory" 2

Herman Geuvers

February 26, 2008

On first order dependent type theory

NB $\rightarrow$ binds strongest.

1. (basic) Find a term of the following type and write down the contexts in which these terms are typed.

$$(\Pi x{:}A.P\,x{\rightarrow}Q\,x){\rightarrow}(\Pi x{:}A.P\,x){\rightarrow}\Pi x{:}A.Q\,x$$

2. (basic) Find a term of the following type and write down the contexts in which these terms are typed.

$$(\Pi x{:}A.P\,x{\rightarrow}\Pi z.R\,z\,z){\rightarrow}(\Pi x{:}A.P\,x){\rightarrow}\Pi z{:}A.R\,z\,z).$$

3. (medium) Construct a term of type $(A(BA))$ in the context for propositonal logic in LF.

4. (medium) Construct a proof-term that mirrors the (obvious) proof of

$$\forall x(P\,xQ\,x)\forall x.P\,x\forall x.Q\,x$$

in the context for predicate logic in LF.

On higher order logic

1. (medium) Prove reflexivity, transitivity and symmetry of $=_A$ (the definable Leibniz equality in HOL).

2. (medium) The transitive closure of a binary relation R on A has been defined as follows.

$$\mathrm{trclos}\,R \quad := \quad \lambda x, y{:}A.$$
$$(\forall Q{:}A{\rightarrow}A{\rightarrow}.((Q){\rightarrow}(R \subseteq Q){\rightarrow}(Q\,x\,y))).$$

   (a) Prove that the transitive closure is transitive.
   (b) Prove that the transitive closure of R contains R.