Type Theory exercises

Exercises 1: Simple Type Theory

Solution to exercise 4.e

Compute the pt of **SKSI**. We first compute the pts of the components.

$$\begin{array}{rcl}
\mathbf{S} & : & (\alpha\to\beta\to\gamma)\to(\alpha\to\beta)\to\alpha\to\gamma \\
\mathbf{K} & : & (\alpha_1\to\beta_1\to\alpha_1) \text{ call this } \sigma_1 \\
\mathbf{S} & : & (\alpha_2\to\beta_2\to\gamma_2)\to(\alpha_2\to\beta_2)\to\alpha_2\to\gamma_2 \text{ call this } \sigma_2 \\
\mathbf{I} & : & \alpha_3\to\alpha_3 \text{ call this } \sigma_3
\end{array}$$

The application **SKSI** yields the following unification problems.

I $\sigma_1 \sim \alpha\to\beta\to\gamma$,

II $\sigma_2 \sim \alpha\to\beta$,

III $\sigma_3 \sim \alpha$.

This again results in the following unification problems

I $\alpha_1 \sim \alpha$, $\beta_1 \sim \beta$, $\alpha_1 \sim \gamma$,

II $\alpha \sim \alpha_2\to\beta_2\to\gamma_2$, $\beta \sim (\alpha_2\to\beta_2)\to\alpha_2\to\gamma_2$,

III $\alpha \sim \alpha_3\to\alpha_3$.

This again yields

$$\alpha \sim \alpha_1 \sim \gamma \sim \alpha_3\to\alpha_3 \sim \alpha_2\to\beta_2\to\gamma_2$$

and

$$\beta \sim \beta_1 \sim (\alpha_2\to\beta_2)\to\alpha_2\to\gamma_2$$

This only yields additional constraints on α_3 and α_2, and we find as a most general unifier the substitution S with

$$\begin{array}{rcl}
S(\alpha) = S(\alpha_1) = S(\gamma) & = & (\beta_2\to\gamma_2)\to\beta_2\to\gamma_2 \\
S(\alpha_2) = S(\alpha_3) & = & \beta_2\to\gamma_2 \\
S(\beta) = S(\beta_1) & = & (\alpha_2\to\beta_2)\to\alpha_2\to\gamma_2
\end{array}$$

This gives us as the pt of **SKSI**:

$$\mathbf{SKSI} : (\beta_2\to\gamma_2)\to\beta_2\to\gamma_2$$

Note that $\mathbf{SKSI} \longrightarrow_\beta \mathbf{I}$, which has a more general pt.