Formal languages, grammars, and automata

Assignment 1, Wednesday, Nov. 12, 2014

Exercise teachers. Recall the following split-up of students:

teacher	lecture room	email	students
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The delivery boxes are located in the Mercator 1 building on the ground floor (where the Computer Science department ICIS is located).

Handing in your answers. The exercises marked with **points** should be handed in:

1. Delivery box (default): Put your solutions in the appropriate delivery box (see above). Before putting your solutions in the box make sure:
   - your name and student number are written clearly on the document.
2. E-mail (in case your exercise class teacher approves): Send your solutions by e-mail to your exercise class teacher (see above) with subject '*assignment 1*'. This e-mail should only contain a single PDF document as attachment. Make sure:
   - the file is a PDF document that is well readable
   - your name is part of the filename (for example MyName_assignment-1.pdf)
   - your name and student number are in the document (since they will be printed).

Deadline: Monday, November 17, 16:00 sharp!

Goals: After completing these exercises successfully you should be able to find a regular expression for a given language, you should be able to determine whether two descriptions of languages determine the same language, you should be able to prove simple properties of words and languages. The total number of points is 20.

1. **(5 points)** Consider the following languages over $\Sigma = \{a, b\}$: $L_1 = \mathcal{L}((a + b)^*)$, $L_2 = \mathcal{L}((a^*b^*)^*)$, and $L_3 = \mathcal{L}(((ab^*)^*))$. Prove that precisely two of these languages are equal.
2. Consider the languages $L_1 = \mathcal{L}((abba)^*)$, $L_2 = \mathcal{L}(a(bba)^*)$, $L_3 = \mathcal{L}((a(bba)^*)^*)$.
   (a) Show that each of these languages is different.
   (b) For which pairs L_i, L_j (with $i \neq j$) doe we have $L_i \subseteq L_j$? Prove your answer.
   (c) For which of these languages do we have $L_k\, L_k \neq L_k$? Prove your answer.
3. Give a regular expression for the following languages and explain your answer.
   (a) **(5 points)**
   $$\{w \in \{a, b, c\}^* \mid |w| \geq 3\}.$$
   (b) **(5 points)**
   $$\{w \in \{a, b\}^* \mid w \text{ begins with } b \text{ and } |w|_b \text{ is even}\}.$$
   (c) **(5 points)**
   $$\{w \in \{a, b\}^* \mid bb \text{ doesn't occur in } w\}.$$
4. [These exercise are hard now. They show that the subject is non-trivial. Later we will learn methods to solve this more easily]

(a) Show that the language

$$\{w \in \{a, b\}^* \mid aa \text{ occurs exactly twice in } w\}.$$

is regular.
[Hint. Beware of the string aaa!]

(b) Show that the language

$$\{w \in \{a, b\}^* \mid |w|_a \text{ and } |w|_b \text{ are even}\}$$

is regular.