Formal languages, grammars, and automata

Assignment 2, Wednesday, Nov. 19, 2014

Exercise teachers. Recall the following split-up of students:

teacher	lecture room	email	students
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The delivery boxes are located in the Mercator 1 building on the ground floor (where the Computer Science department ICIS is located).

Handing in your answers. The exercises marked with **points** should be handed in:

1. Delivery box (default): Put your solutions in the appropriate delivery box (see above). Before putting your solutions in the box make sure:
   - your name and student number are written clearly on the document.
2. E-mail (in case your exercise class teacher approves): Send your solutions by e-mail to your exercise class teacher (see above) with subject '*assignment 2*'. This e-mail should only contain a single PDF document as attachment. Make sure:
   - the file is a PDF document that is well readable
   - your name is part of the filename (for example MyName_assignment-2.pdf)
   - your name and student number are in the document (since they will be printed).

Deadline: Monday, November 24, 16:00 sharp!
Goals: After completing these exercises successfully you should be able to make a state diagram for a DFA and check whether words are accepted by the DFA; you should be able to construct a DFA for a given language and you should be able to compute a regular expression for a DFA. The total number of points is 20.

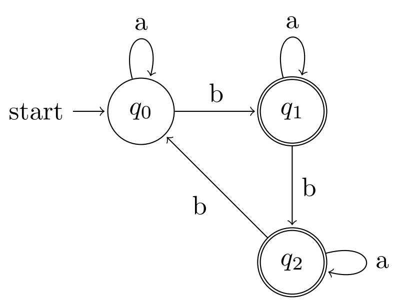
1. Let $M = (Q, q_0, \delta, F)$ be the deterministic finite automaton (DFA) over $\Sigma = \{a, b, c\}$, where $Q = \{q_0, q_A, q_B, q_C\}$, $F = \{q_0\}$ and δ is given by the table

δ	a	b	c
q_0	q_A	q_B	q_C
q_A	q_0	q_C	q_B
q_B	q_C	q_0	q_A
q_C	q_B	q_A	q_0

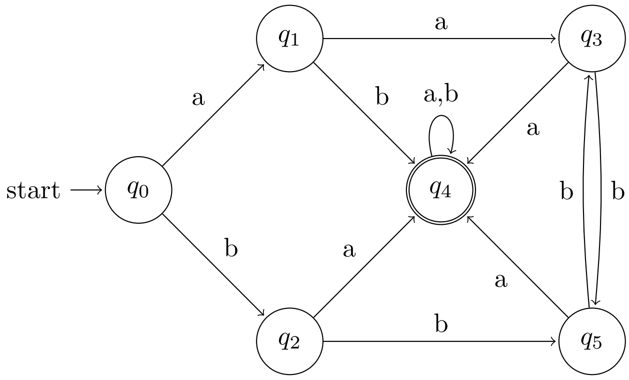
   (a) Make a state transition diagram for M.
   (b) Determine for the following words whether they belong to $\mathcal{L}(M)$: $abba, baab, bac, cac$.
   (c) Is it the case that $\{a^n b^n | n \geq 0\} \subseteq \mathcal{L}(M)$? Give a proof or a counterexample.
   (d) Is it the case that $\{a^n c^n b^n | n \geq 0\} \subseteq \mathcal{L}(M)$? Give a proof or a counterexample.

2. Let $\Sigma = \{a, b\}$. (Give DFAs by drawing their state diagrams.)
   (a) **(5 points)** Give a DFA that accepts $L_1 := \{w \in \Sigma^* \mid w \text{ does not contain } ab\}$, and prove that your answer is correct.
   (b) **(5 points)** Give a DFA that accepts $L_2 := \{w \in \Sigma^* \mid |w|_a \text{ is even and } |w|_b \text{ is odd}\}$ and prove that your answer is correct.
   (c) **(5 points)** Give a DFA that accepts $L_3 := \{w \in \Sigma^* \mid w \text{ contains } aa \text{ twice}\}$. (Beware of aaa.)

3. **(5 points)** Consider the DFA M below and compute a regular expression e such that $\mathcal{L}(M) = \mathcal{L}(e)$, using the method that was shown at the lecture (and that is also in Section 4.2.2 of the lecture notes of Silva). Show the steps in your computation.



4. Let a DFA M over $\Sigma = \{a, b\}$, be given by



Show that the set $\{w \in \Sigma^* | w \text{ ends with } bb\} \cap \mathcal{L}(M)$ is regular by giving a regular expression for it.

5. (a) [More challenging question] Give a deterministic finite state automaton (DFA) with 7 states that accepts $\mathcal{L}((a+b)^*(aba + baab)(a+b)^*)$.

   (b) [Challenge question] Let M be some finite state automaton that accepts the language $\mathcal{L}(M)$. Show that

$$L' := \{w | w \text{ is a prefix of some word } v \in \mathcal{L}(M)\}$$

   can also be accepted by some finite state automaton.
   (NB: w is a *prefix* of v if $wu = v$ for some word u. Phrased differently: w is a *beginpart* of v.)