Formal languages, grammars, and automata

Assignment 3, Wednesday, Nov. 26, 2014

Exercise teachers. Recall the following split-up of students:

teacher	**lecture room**	**email**	**students**
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The delivery boxes are located in the Mercator 1 building on the ground floor (where the Computer Science department ICIS is located).

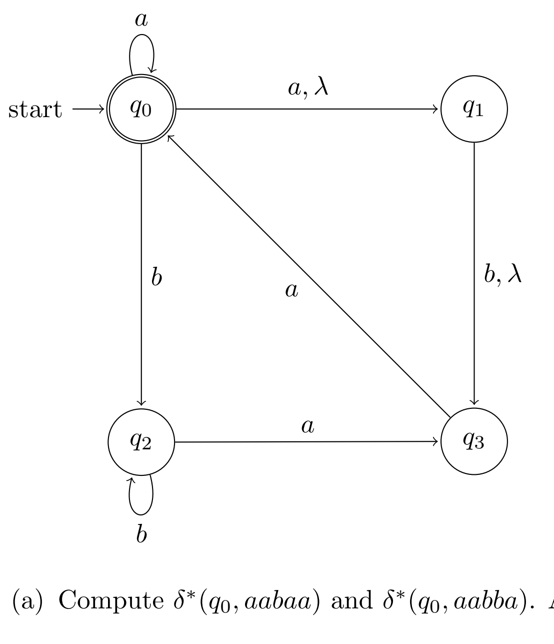
Handing in your answers. The exercises marked with **points** should be handed in:

1. Delivery box (default): Put your solutions in the appropriate delivery box (see above). Before putting your solutions in the box make sure:
   - your name and student number are written clearly on the document.
2. E-mail (in case your exercise class teacher approves): Send your solutions by e-mail to your exercise class teacher (see above) with subject '*assignment 3*'. This e-mail should only contain a single PDF document as attachment. Make sure:
   - the file is a PDF document that is well readable
   - your name is part of the filename (for example MyName_assignment-3.pdf)
   - your name and student number are in the document (since they will be printed).

Deadline: Monday, December 1, 16:00 sharp!

Goals: After completing these exercises successfully you should be able to "read" a state diagram for an NFA (and NFA$_\lambda$) and check whether words are accepted by the NFA (resp. NFA$_\lambda$); you should be able to construct an NFA (NFA$_\lambda$) for a given language and you should be able to eliminate λ-steps and non-determinism from an NFA (or NFA$_\lambda$) to compute an equivalent DFA; you should be able to construct an NFA$_\lambda$ for a regular expression. The total number of points is 20.

1. Consider the following NFA$_\lambda$, M.



   (a) Compute $\delta^*(q_0, aabaa)$ and $\delta^*(q_0, aabba)$. Are these words accepted?

(b) Which of the following words are accepted by M: $aabbaa$, bbb? Explain.

(c) Compute λ-closure(q_0), λ-closure(q_1), λ-closure(q_2) and λ-closure(q_3).

(d) Eliminate non-determinism and λ-steps (using the subset construction) and compute the associated DFA M'.

2. In a number of steps, we construct a regular expression e such that

$$\mathcal{L}(e) = L = \{w \in \Sigma^* \mid abba \text{ is not a substring of } w\}.$$

(a) **(5 points)** Give an NFA M such that its language is $\overline{L}$. (NB: This is the *complement* of L.) Explain your answer.

(b) **(5 points)** Construct from M, using the subset construction, a DFA M' accepting the same language. Reduce the number of states in M' by joining states that have the same effect. This yields a DFA M''.

(c) Modify M'' to obtain M''' that accepts L.

(d) Compute a regular expression e such that $\mathcal{L}(e) = \mathcal{L}(M''')$.

3. Consider the regular expression $e = ((aa + b)^* ba)^*$.

(a) **(5 points)** Construct an NFA$_\lambda$ M that accepts the language $\mathcal{L}(e)$, following the steps shown in the lecture (or the steps of the course notes LnA). Explain what you are doing.

(b) **(5 points)** Construct a DFA M' out of M that accepts the same language $\mathcal{L}(e)$. Explain what you are doing.

4. (More challenging) Let L over $\Sigma = \{a, b\}$ be regular. Show that the following language L' is also regular.

$$L' := \{w \in \Sigma^* \mid \exists v \in L (w \text{ is contained in } v)\}$$

NB. w is *contained in* v if all the symbols of w occur in v in the same order, to be precise: $v = s_1 \dots s_n$ and $w = s_{i_1} \dots s_{i_m}$ for some sequence $1 \leq i_1 < \dots < i_m \leq n$. ($s_j \in \Sigma$.)