Formal languages, grammars, and automata

Assignment 5, Wednesday, Dec. 17 2014

Exercises with answers

1. Let $\Sigma = \{a, b\}$. Consider the context-free grammars

$G_1 =$

S	→	aABb
A	→	aA | a
B	→	bB | b

and

$G_2 =$

S	→	AABB
A	→	AA | a
B	→	BB | b

   (a) Describe $L_1 = \mathcal{L}(G_1)$ and $L_2 = \mathcal{L}(G_2)$.

   (b) Show that $L_1 = L_2$.
   Hint: first show that A in G_1 generates the same language as A in G_2, etc.

   (c) Is G_1 ambiguous? Is G_2 ambiguous? If so, give a word and two different leftmost derivations.

   (d) Give a derivation of $aaaabbb$ in G_1 and in G_2, and draw the corresponding derivation trees.

2. For each of the following languages construct a context-free grammar that generates the language, and explain why your answer is correct.

(5 points) $L_3 = \{a^n b^{n+m} a^m \mid n, m \geq 0\}$
(5 points) $L_4 = \{w \in \{a, b\}^* \mid |w|_a \text{ is even}\}$
(5 points) $L_5 = \mathcal{L}((ab)^*(a + bb)^*)$
$L_6 = \{w \in \{a, b\}^* \mid |w| = 2k + 1 \text{ and } w_1 = w_{k+1}\}$

where w_i denotes the i-th symbol in a word w. That is, L_6 consists of all words of odd length that have the same symbol in the first and middle positions.

Answer Info: ....

(a) For L_3:

$G_3 =$

S	→	$T\ U$
T	→	$\lambda \mid aTb$
U	→	$\lambda \mid bUa$

Explanation: S produces words of the form $a^n b^n b^m a^m$, so exactly the words in L_3, because T produces words of the form $a^n b^n$ and U produces words of the form $b^m a^m$.

(b) For L_4:

$G_4 =$

S	→	$B \mid BaSaB$
B	→	$bB \mid \lambda$

Explanation: B produces an arbitrary number of b's: b^n for $n \geq 0$. S produces, consecutively, the outermost pair of a's in $w \in L_4$ and then the next-to outermost pair of a's in $w \in L_4$ etcetera. In between these a's an arbitarry number of b's can be written.

(c) *For L_5:*

$$G_5 = \boxed{\begin{array}{lcl} S & \to & T\ U \\ T & \to & \lambda \mid abT \\ U & \to & \lambda \mid bbU \mid aU \end{array}}$$

Explanation: T produces $\mathcal{L}((ab)^*)$; U produces $\mathcal{L}((a+bb)^*)$, so S produces L_5.

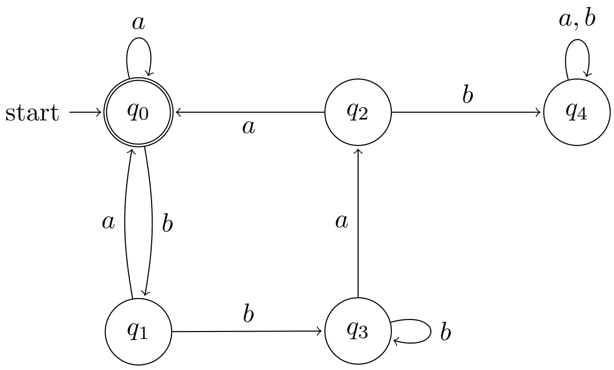
(d) *For L_6:*

$$G_6 = \boxed{\begin{array}{lcl} S & \to & aA \mid bB \mid a \mid b \\ A & \to & aa \mid ab \mid aAa \mid aAb \mid bAb \mid bAa \\ B & \to & ba \mid bb \mid aBa \mid aBb \mid bBb \mid bBa \end{array}}$$

Explanation: A produces a word of positive even length $2k$ (so $k \geq 1$) with an a in the k-th position: if $A \Rightarrow w$, then $w = x_1 \dots x_{k-1} a y_1 \dots y_k$, with $x_i, y_i \in \{a, b\}$, $k \geq 1$. Similarly, B produces a word of positive even length $2k$ with a b in the k-th position. So, the first two rules for S produces a word of odd length $2k+1$ with $k \geq 1$ and the same symbol at position 1 and $k+1$. The last two rules for S extend this to all $k \geq 0$.

End Answer Info . . . .

3. **(5 points)** Consider the following DFA M,



Construct a context-free grammar that generates $\mathcal{L}(M)$.
Answer Info: . . . .
Using the procedure discussed on the slides:

$$G = \boxed{\begin{array}{lcl} S & \to & aS \mid bA \mid \lambda \\ A & \to & aS \mid bT \\ B & \to & aS \mid bU \\ T & \to & aB \mid bT \\ U & \to & aU \mid bU \end{array}}$$

End Answer Info . . . .

4. $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, \times, +, (,)\}$. Construct a context-free grammar that generates the language

$$L_7 = \{w \in \Sigma^* \mid w \text{ is a well-formed arithmetical expressions}\}$$

NB. $2+3+4\times 5$ and $((2+3)+4)\times 5$ and $(((2+3)))+4\times 5$ are well-formed. $2+(3+4\times 5$ and $(2+3)+4)\times 5$ and $)($ are not.

5. Let $G_8 = \langle V_8, S_8, P_8 \rangle$ be a context-free grammar with start symbol S_8. Construct a context-free grammar for the language $\mathcal{L}(G_8)^*$.