Formal languages, grammars, and automata

Assignment 5, Wednesday, Dec. 17 2014

Exercise teachers. Recall the following split-up of students:

teacher	lecture room	email	students
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The delivery boxes are located in the Mercator 1 building on the ground floor (where the Computer Science department ICIS is located).

Handing in your answers. The exercises marked with **points** should be handed in:

1. Delivery box (default): Put your solutions in the appropriate delivery box (see above). Before putting your solutions in the box make sure:
   - your name and student number are written clearly on the document.
2. E-mail (in case your exercise class teacher approves): Send your solutions by e-mail to your exercise class teacher (see above) with subject '*assignment 5*'. This e-mail should only contain a single PDF document as attachment. Make sure:
   - the file is a PDF document that is well readable
   - your name is part of the filename (for example MyName_assignment-5.pdf)
   - your name and student number are in the document (since they will be printed).

Deadline: Monday, January 5, 16:00 sharp!

Goals: After completing these exercises successfully you should be able to read context-free grammars, as well as write down grammars for context-free languages and regular languages. The total number of points is 20.

1. Let $\Sigma = \{a, b\}$. Consider the context-free grammars

$$G_1 = \boxed{\begin{array}{lcl} \mathrm{S} & \to & \mathrm{aABb} \\ \mathrm{A} & \to & \mathrm{aA} \mid \mathrm{a} \\ \mathrm{B} & \to & \mathrm{bB} \mid \mathrm{b} \end{array}} \quad \text{and} \quad G_2 = \boxed{\begin{array}{lcl} \mathrm{S} & \to & \mathrm{AABB} \\ \mathrm{A} & \to & \mathrm{AA} \mid \mathrm{a} \\ \mathrm{B} & \to & \mathrm{BB} \mid \mathrm{b} \end{array}}$$

   (a) Describe $L_1 = \mathcal{L}(G_1)$ and $L_2 = \mathcal{L}(G_2)$.

   (b) Show that $L_1 = L_2$.
   Hint: first show that A in G_1 generates the same language as A in G_2, etc.

   (c) Is G_1 ambiguous? Is G_2 ambiguous? If so, give a word and two different leftmost derivations.

   (d) Give a derivation of $aaaabbb$ in G_1 and in G_2, and draw the corresponding derivation trees.

2. For each of the following languages construct a context-free grammar that generates the language, and explain why your answer is correct.

   (5 points) $L_3 = \{a^n b^{n+m} a^m \mid n, m \geq 0\}$

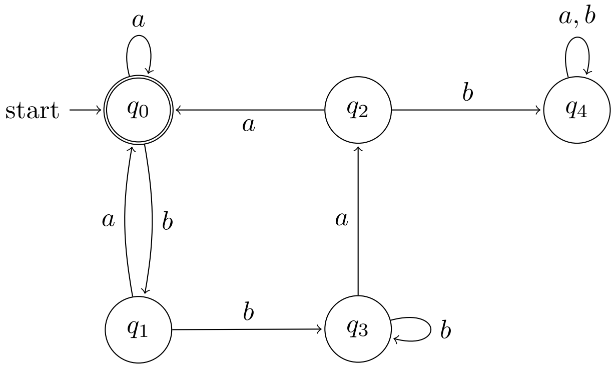
   (5 points) $L_4 = \{w \in \{a, b\}^* \mid |w|_a \text{ is even}\}$

   (5 points) $L_5 = \mathcal{L}((ab)^*(a + bb)^*)$

   $L_6 = \{w \in \{a, b\}^* \mid |w| = 2k + 1 \text{ and } w_1 = w_{k+1}\}$

   where w_i denotes the i-th symbol in a word w. That is, L_6 consists of all words of odd length that have the same symbol in the first and middle positions.

3. **(5 points)** Consider the following DFA M,



Construct a context-free grammar that generates $\mathcal{L}(M)$.

4. $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, \times, +, (,)\}$. Construct a context-free grammar that generates the language

$$L_7 = \{w \in \Sigma^* \mid w \text{ is a well-formed arithmetical expressions}\}$$

NB. $2+3+4\times 5$ and $((2+3)+4)\times 5$ and $(((2+3)))+4\times 5$ are well-formed. $2+(3+4\times 5$ and $(2+3)+4)\times 5$ and $)($ are not.

5. Let $G_8 = \langle V_8, S_8, P_8 \rangle$ be a context-free grammar with start symbol S_8. Construct a context-free grammar for the language $\mathcal{L}(G_8)^*$.