Formal languages, grammars, and automata

Assignment 6, Wednesday, Jan. 7 2015

Exercises with answers

1. For each of the following languages L_i, construct a PDA M_i that accepts L_i, $i = 1, 2, 3, 4$. In each case, explain why the constructed PDA accepts that language.

   (a) **(5 points)** $L_1 = \{a^n b^{2n} \in \{a,b\}^* \mid n \geq 0\}$.

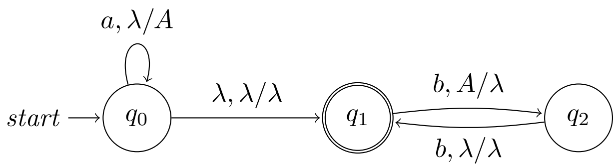
   (b) **(5 points)** $L_2 = \{w \in \{a,b\}^* \mid w = w^R\}$.

   (c) $L_3 = \{w \in \{a,b\}^* \mid |w|_a = |w|_b\}$.

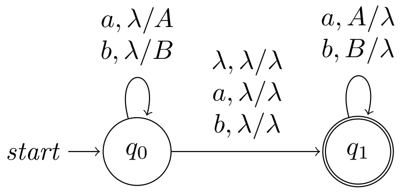
   (d) **(5 points)** $L_4 = \{w \in \{a,b\}^* \mid |w|_a \leq |w|_b\}$.

Answer Info: ....

(a)

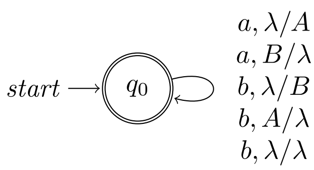


(b)



(c)

(d)



End Answer Info ....

2. Let $M = \langle Q, \Sigma, \Gamma, \delta, q_0, F \rangle$ be the PDA given by:

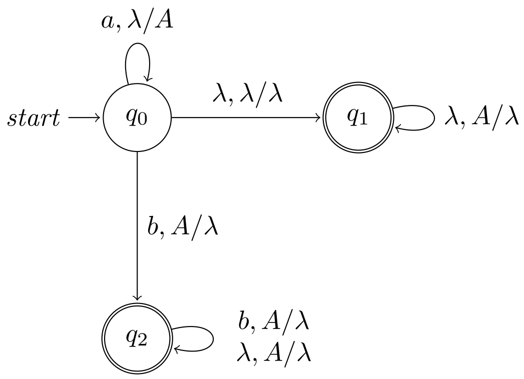
$$\begin{array}{rcl} Q &=& \{q_0, q_1, q_2\} \\ \Sigma &=& \{a, b\} \\ \Gamma &=& \{A\} \\ F &=& \{q_1, q_2\} \end{array} \qquad \begin{array}{rcl} \delta(q_0, a, \lambda) &=& \{\langle q_0, A\rangle\} \\ \delta(q_0, \lambda, \lambda) &=& \{\langle q_1, \lambda\rangle\} \\ \delta(q_0, b, A) &=& \{\langle q_2, \lambda\rangle\} \\ \delta(q_1, \lambda, A) &=& \{\langle q_1, \lambda\rangle\} \\ \delta(q_2, b, A) &=& \{\langle q_2, \lambda\rangle\} \\ \delta(q_2, \lambda, A) &=& \{\langle q_2, \lambda\rangle\} \end{array}$$

   (a) **(5 points)** Draw the state diagram of M.

(b) **(5 points)** For each of the following words, check whether they are accepted by M, and if so, give a successful computation, if not, give all non-accepting computations: $abb, aba, aabb, aaab$.

(c) **(5 points)** Describe the language accepted by M using set comprehension notation.

Answer Info: . . . .

(a) 

(b) *For abb:*
$\langle q_0, abb, \lambda\rangle \to \langle q_0, bb, A\rangle \to \langle q_2, b, \lambda\rangle$,
$\langle q_0, abb, \lambda\rangle \to \langle q_1, abb, \lambda\rangle$,
this word is not accepted.

For aba:
$\langle q_0, aba, \lambda\rangle \to \langle q_0, ba, A\rangle \to \langle q_2, a, \lambda\rangle$,
$\langle q_0, aba, \lambda\rangle \to \langle q_1, aba, \lambda\rangle$, $\langle q_0, aba, \lambda\rangle \to \langle q_0, ba, A\rangle \to \langle q_1, ba, A\rangle \to \langle q_1, ba, \lambda\rangle$,
this word is not accepted.

For aabb:
$\langle q_0, aabb, \lambda\rangle \to \langle q_0, abb, A\rangle \to \langle q_0, bb, AA\rangle \to \langle q_2, b, A\rangle \to \langle q_2, \lambda, \lambda\rangle$,
this word is accepted.

For aaab:
$\langle q_0, aaab, \lambda\rangle \to \langle q_0, aab, A\rangle \to \langle q_0, ab, AA\rangle \to \langle q_0, b, AAA\rangle \to \langle q_2, \lambda, AA\rangle \to \langle q_2, \lambda, A\rangle \to \langle q_2, \lambda, \lambda\rangle$,
this word is accepted.

(c) $\mathcal{L}(M) = \{a^n b^m \mid n \geq m\}$

End Answer Info . . . .

3. Consider the following context free grammar G_3

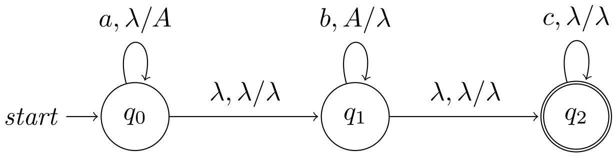
S	$\to$	AC
A	$\to$	$aAB \mid \lambda$
B	$\to$	b
C	$\to$	$cC \mid \lambda$

(a) **(5 points)** Describe the language $\mathcal{L}(G_3)$.

(b) **(5 points)** Construct a PDA that accepts $\mathcal{L}(G_3)$.

Answer Info: . . . .

(a) $\mathcal{L}(G_3) = \{a^n b^n c^m \mid m, n \geq 0\}$.

(b) 

End Answer Info ....

4. (Extra exercise) Construct a *queue* automaton that accepts the language $\{a^n b^n c^n \mid n \geq 0\}$. (A queue automaton is a PDA that has a queue instead of a stack)