1. Consider the following regular expressions
   \[ r_1 = a(ba)^*b \quad r_2 = (a(ba)^*b)^* \quad r_3 = (ab)^*ab \quad r_4 = (ab)^* \]
   
   (a) Give 2 regular expressions \( r_i \) and \( r_j \) \((i \neq j)\) such that \( L(r_i) = L(r_j) \). Show that your answer is correct.
   
   (b) Give 2 regular expressions \( r_i \) and \( r_j \) such that \( L(r_i) \neq L(r_j) \). Show that your answer is correct.

2. Consider the language
   \[ L := \{ w \in \{a,b\}^* \mid \#_a(w) \text{ is divisible by } 3 \} \]
   
   NB “divisible by 3” is in Dutch: “deelbaar door 3”
   
   (a) Give a regular expression \( e \) such that \( L(e) = L \).
   
   (b) Give a regular grammar \( G \) such that \( L(G) = L \).

3. Consider the following NFA (non-deterministic finite automaton) \( M \)

   (a) Indicate for each of the following words whether they are accepted by \( M \): \( abba, ababa, abab \). Explain your answer.
   
   (b) Give a regular expression \( e \) such that \( L(e) = L(M) \). Explain your answer.
   
   (c) Construct a DFA \( M' \) that accepts the same language as \( M \).

Continue on other side
4. Give a DFA (deterministic finite automaton) that accepts the language $L_2$ where

$$L_2 = \{ w \in \{a,b\}^* \mid \#_a(w) \text{ is even and } w \text{ does not end with } aa\}.$$ 

5. Consider the language over $\{a,b,c\}$

$$L := \{cwcv\mid w,v \in \{a,b\}^* \text{ and } |w| = |v|\}$$

So, $w$ and $v$ should be of equal length and not contain the symbol $c$.

(a) Give a context-free grammar $G$ such that $L = L(G)$.

(b) Give a pushdown automaton $M$ such that $L = L(M)$.

(c) Prove that $L$ is not regular.

6. Let $M$ be the PDA with

$$Q = \{q_0, q_1\} \quad \delta(q_0, a, \lambda) = \{[q_0, A]\}$$

$$\Sigma = \{a, b, c\} \quad \delta(q_0, b, \lambda) = \{[q_0, B]\}$$

$$\Gamma = \{A, B\} \quad \delta(q_0, \lambda, A) = \{[q_1, \lambda]\}$$

$$F = \{q_1\} \quad \delta(q_1, c, A) = \{[q_1, \lambda]\}$$

$$\delta(q_1, \lambda, B) = \{[q_1, \lambda]\}$$

(a) Draw a state diagram for $M$.

(b) Check which of the following words is in $L(M)$ and explain your answer:

- $bba$,
- $abba$,
- $baaacc$.

(c) Is $L(b^*a) \subseteq L(M)$? Explain your answer.

(d) For which $p, m, n \in \mathbb{N}$ do we have $b^p a^m c^n \in L(M)$? Explain your answer.

(e) Describe $L(M)$ using set-notation.

7. (BONUS) Let $\Sigma = \{a,b\}$. Prove that, if $L$ is regular over $\Sigma$, then $\text{Max}(L)$ is also regular over $\Sigma$, where

$$\text{Max}(L) = \{ w \in L \mid \forall v \in \Sigma^* (wv \in L \rightarrow v = \lambda) \}$$

So, $\text{Max}(L)$ consists of the words in $L$ that cannot be ‘extended’.