

(7) (a) $L(r_1) = L(r_3)$ $r_1 = a(ba)^*b$
 $r_3 = (ab)^*ab$

Proof if $w \in L(r_1)$, then $w = a(ba)^n b$ for some $n \geq 0$

so $w = \underbrace{aba \dots aba}_n b$

so $w = \underbrace{ab \dots ab}_n ab$

so $w \in L(r_3)$

if $w \in L(r_3)$, then $w = (ab)^n ab$

so $w = \underbrace{ab \dots ab}_n ab$

so $w = a \underbrace{ba \dots ba}_n b$

so $w \in L(a(ba)^*b)$

(7) (b) $L(r_4) \neq L(r_3)$, because $\lambda \in L(r_4)$

$\lambda \notin L(r_3)$, because a ~~string~~ $w \in L(r_3)$ should contain ab

$L(r_4) \neq L(r_1)$

similar.

$L(r_1) \neq L(r_2)$ { also because $\lambda \in L(r_2)$

$L(r_2) \neq L(r_1)$

NB $L(r_2) = L(r_4)$, but that's a bit harder to show (but not very difficult)

(2) $L = \{w \in \{a,b\}^* \mid \#_a(w) \text{ is divisible by } 3\}$.

(a) $e = b^* (b^* a b^* a b^* a b^*)^* b^*$

(~~the~~ explanation [not required]): a number of b's followed by a number of words with exactly 3 a's followed by a number of b's)

Shorter:

$$e = b^* (a b^* a b^* a b^*)^*$$

$$e = (b^* a b^* a b^* a)^* b^*$$

(b) $S \rightarrow aA \mid bS \mid \lambda$

$A \rightarrow aB \mid bA \mid$

$B \rightarrow aS \mid bB$

S: $\#_a(w)$ is a 3-fold
 A: $\#_a(w)$ is a 1-fold + 1
 B: $\#_a(w)$ is a 1-fold + 2

3 a

- abba is ~~not~~ accepted: (!)

$q_0, abba \vdash q_1, bba \vdash q_2, ba \vdash q_0, a \vdash q_2, \lambda \notin \mathcal{L}$

- ababa is accepted:

$q_0, ababa \vdash q_2, baba \vdash q_0, aba \vdash q_2, ba \vdash q_0, a \vdash q_2, \lambda \in \mathcal{L}$

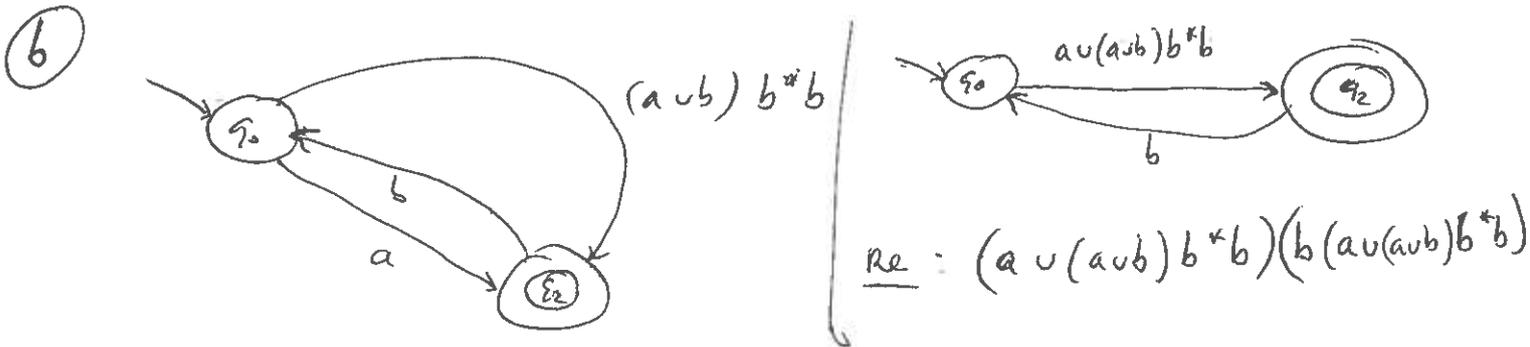
- abab is ~~not~~ accepted (!)

Possible runs:

$q_0, abab \vdash q_1, bab \vdash q_2, ab \text{ stop}$

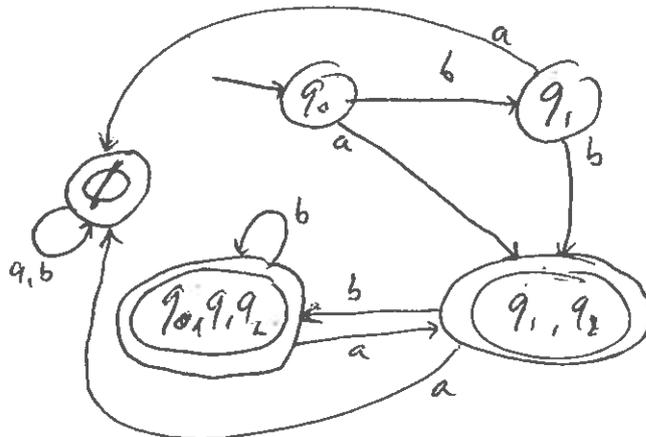
$\vdash q_1, ab \text{ stop}$

$q_0, abab \vdash q_2, bab \vdash q_0, ab \vdash q_1, b \vdash q_2, \lambda \notin \mathcal{L}$

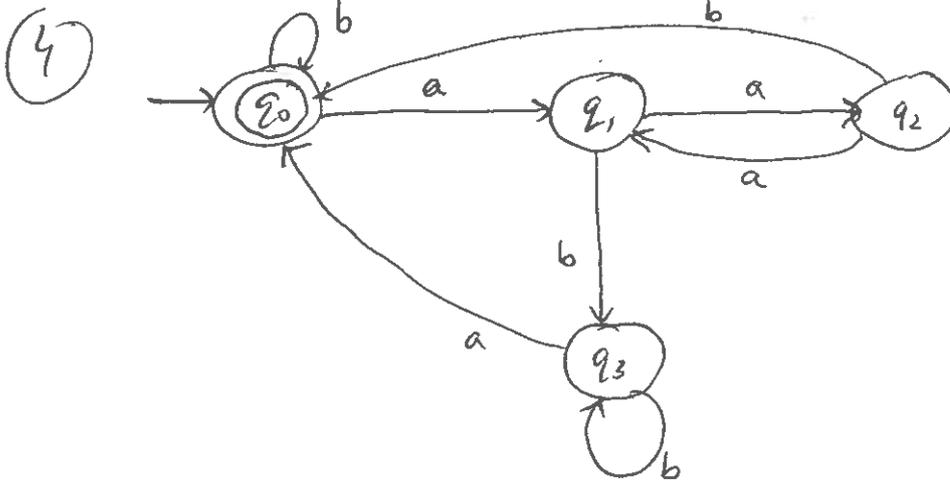


c Table

	a	b
q_0	q_1, q_2	\emptyset, q_1
q_1	\emptyset	q_1, q_2
q_1, q_2	\emptyset	q_0, q_1, q_2
q_0, q_1, q_2	q_1, q_2	q_0, q_1, q_2



NB $RE: (bb^*a)((bb^*a)^* \cup b(b^*ab)^*)$
 $(bb^*a)((bb^*a)^* \cup bb^*(abb^*)^*)$



q_0 : $\#_a(w)$ even, w does not end with aa

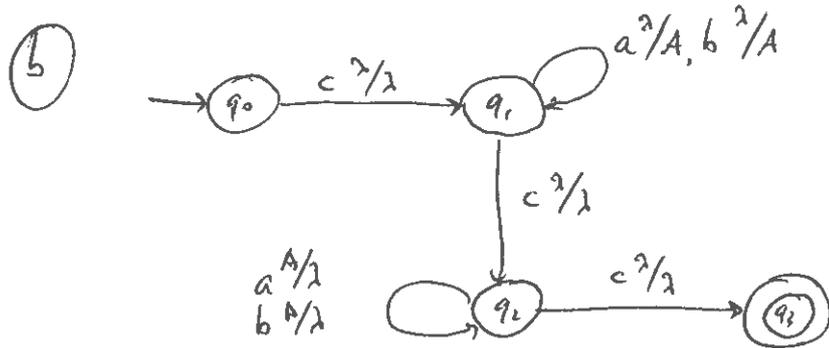
q_1 : $\#_a(w)$ odd, w ends with a .

q_2 : $\#_a(w)$ even, w ends with aa

q_3 : $\#_a(w)$ odd, w ends with b

(5) ~~(1)~~ $L = \{ cwcv^2c \mid w, v \in \{a, b\}^*, |w| = |v|\}$.

(a) $S \rightarrow cXc$
 $X \rightarrow aXa \mid aXb \mid bXa \mid bXb \mid c$



(c) Suppose L is regular
 Let p be as in the Pumping Lemma

Take $w = ca^pca^pc$

Then there are x, y, z such that

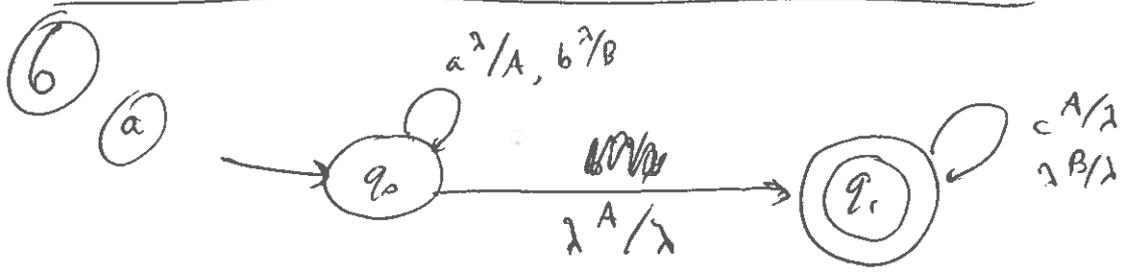
$$w = xyz \text{ and } |xy| \leq p \text{ and } |y| \geq 1 \text{ and } xy^nz \in L \text{ (for } n \in \mathbb{N})$$

$|xy| \leq p$ and $|y| \geq 1$, so either $y = a^q$ for some $q \geq 1$ or $x = \lambda, y = ca^q$ ($q \geq 0$)

Case $(*)$: $xy^nz = ca^{p-2}ca^pc \in L \nrightarrow$ so L not regular
 Case $(**)$: $xy^nz = a^rca^pc \in L \nrightarrow$ so L not regular

So: L not regular

[NB Can also choose $n = 2$]



b. $bb a \in L$

$q_0, bb a, \lambda \vdash q_0, ba, B \vdash q_0, a, BB \vdash q_0, \lambda, ABB \vdash q_1, \lambda, BB$
 $\vdash q_1, \lambda, B \vdash q_1, \lambda, \lambda \quad \S$

$abba \notin L$

$q_0, abba, \lambda \vdash q_0, bba, A \vdash q_1, bba, \lambda \quad \text{stop}$
 $q_0, abba, \lambda \vdash q_0, bba, A \vdash q_0, ba, BA \vdash q_0, a, BBA$
 $\vdash q_0, \lambda, ABBA \vdash q_1, \lambda, BBA \vdash q_1, \lambda, BA \vdash q_1, \lambda, A \quad \text{stop}$

c. $baacc \in L$

$q_0, baacc, \lambda \vdash q_0, aacc, B \vdash \dots q_0, cc, AAB \vdash q_1, cc, AAB$
 $\vdash q_1, c, AB \vdash q_1, \lambda, B \vdash q_1, \lambda, \lambda \quad \S$

c) Yes, $L(b^*a) \subseteq L(M)$

If $w \in L(b^*a)$, then $w = b^n a$ for some $n \geq 0$

This word is accepted by $q_0, b^n a, \lambda \vdash^* q_0, a, B^n \vdash q_0, \lambda, AB^n$
 $\vdash q_1, \lambda, B^n \vdash^* q_1, \lambda, \lambda$

d) $b^p a^m c^n \in L(M)$ if and only if $m \geq 1$ and $n+1 = m$
 $p, n \geq 0$

Proof(\Leftarrow) $b^p a^m c^n$ is accepted by $q_0, b^p a^m c^n, \lambda \vdash^* q_0, c^n, A^m B^p \vdash q_1, c^n, A^m B^p \vdash^* q_1, \lambda, B^p \vdash^* q_1, \lambda, \lambda$

~~RAA~~

(6d) Continued

\Rightarrow If $b^p a^m c^n \in L(A)$, then all b and a are read in q_0
all c are read in q_1

Reading a adds A , reading b adds B to the stack,

• so the last read symbol in $b^p a^m$ should be a .

• The number of c 's should be one ~~more~~ less than the number of a 's,
(because λ also pops a from the stack)

So $m \geq 1$ and $n = m - 1$

(7) ~~Suppose~~ Let L be a regular language and suppose that M is a DFA that accepts L , say $M = (Q, \Sigma, q_0, \delta, F)$

If q_1 is a final state such that $(q_1) \xrightarrow{w} (q_2)$, for some $w \neq \lambda, q_2 \in Q$

then all words that are accepted in q_1 should not be in $\text{Max}(L)$

So we remove from F all states q_i for which $\exists w \neq \lambda \exists q_2 \in F ((q_i, w) \xrightarrow{*} (q_2, \lambda))$

then obtain F'

Then $M' = (Q, \Sigma, q_0, \delta, F')$ accepts $\text{Max}(L)$