Huygens College Reflection
Assignment 5, Wednesday, Nov. 16 2015

1. Give a sentence (actually a 'word') in the language of the 'John & Jill' example grammar (Lecture Notes 2.20). Also give a sentence that is *not* in the grammar, but that is correct English and uses the same alfabet. What is that alphabet?

2. Let $\Sigma = \{a, b\}$. Consider the context-free grammars

$$G_1 = \begin{array}{|lll|} \hline S & \rightarrow & aABb \\ A & \rightarrow & aA \mid a \\ B & \rightarrow & bB \mid b \\ \hline \end{array} \qquad \text{and} \qquad G_2 = \begin{array}{|lll|} \hline S & \rightarrow & AABB \\ A & \rightarrow & AA \mid a \\ B & \rightarrow & BB \mid b \\ \hline \end{array}$$

   (a) Give a derivation of *aaaabbb* in G_1 and in G_2, and draw the corresponding derivation trees.

   (b) Is G_1 ambiguous? Is G_2 ambiguous? If so, give a word and two different leftmost derivations.

   (c) Describe $L_1 = \mathcal{L}(G_1)$ and $L_2 = \mathcal{L}(G_2)$.

   (d) Show that $L_1 = L_2$.
   Hint: first show that A in G_1 generates the same language as A in G_2, etc.

3. For each of the following languages construct a context-free grammar that generates the language, and explain why your answer is correct. (In the first two cases, the language is regular; can you make a grammar that is right-linear?)

$$L_3 = \{w \in \{a, b\}^* \mid |w|_a \text{ is even}\}$$
$$L_4 = \mathcal{L}((ab)^*(a + bb)^*)$$
$$L_5 = \{a^n b^{n+m} a^m \mid n, m \geq 0\}$$