Huygens College Reflection

Assignment 5, Wednesday, Nov. 16 2015

1. Give a sentence (actually a 'word') in the language of the 'John & Jill' example grammar (Lecture Notes 2.20). Also give a sentence that is *not* in the grammar, but that is correct English and uses the same alfabet. What is that alphabet?

2. Let $\Sigma = \{a, b\}$. Consider the context-free grammars

$G_1 =$

S	→	aABb
A	→	aA | a
B	→	bB | b

and

$G_2 =$

S	→	AABB
A	→	AA | a
B	→	BB | b

   (a) Give a derivation of *aaaabbb* in G_1 and in G_2, and draw the corresponding derivation trees.

   (b) Is G_1 ambiguous? Is G_2 ambiguous? If so, give a word and two different leftmost derivations.

   (c) Describe $L_1 = \mathcal{L}(G_1)$ and $L_2 = \mathcal{L}(G_2)$.

   (d) Show that $L_1 = L_2$.
   Hint: first show that A in G_1 generates the same language as A in G_2, etc.

3. For each of the following languages construct a context-free grammar that generates the language, and explain why your answer is correct. (In the first two cases, the language is regular; can you make a grammar that is right-linear?)

$$L_3 = \{w \in \{a,b\}^* \mid |w|_a \text{ is even}\}$$
$$L_4 = \mathcal{L}((ab)^*(a + bb)^*)$$
$$L_5 = \{a^n b^{n+m} a^m \mid n, m \geq 0\}$$

Answer Info: ....

(a) For L_3:

$G_4 =$

S	→	$B \mid BaSaB$
B	→	$bB \mid \lambda$

Explanation: B produces an arbitrary number of b's: b^n for $n \geq 0$. S produces, consecutively, the outermost pair of a's in $w \in L_4$ and then the next-to outermost pair of a's in $w \in L_4$ etcetera. In between these a's an arbitarry number of b's can be written.
In right-linear form (check!):

$G_4 =$

S	→	$aA \mid bS \mid \lambda$
A	→	$aS \mid bA$

(b) For L_4:

$$G_5 = \boxed{\begin{array}{lll} S & \to & T\ U \\ T & \to & \lambda \mid abT \\ U & \to & \lambda \mid bbU \mid aU \end{array}}$$

Explanation: T produces $\mathcal{L}((ab)^*)$; U produces $\mathcal{L}((a+bb)^*)$, so S produces L_5.
In right-linear form (check!):

$$G_5 = \boxed{\begin{array}{lll} S & \to & aA \mid aB \mid bC \mid \lambda \\ A & \to & bS \\ B & \to & aB \mid bC \mid \lambda \\ C & \to & bB \end{array}}$$

(c) For L_5:

$$G_3 = \boxed{\begin{array}{lll} S & \to & T\ U \\ T & \to & \lambda \mid aTb \\ U & \to & \lambda \mid bUa \end{array}}$$

Explanation: S produces words of the form $a^n b^n b^m a^m$, so exactly the words in L_3, because T produces words of the form $a^n b^n$ and U produces words of the form $b^m a^m$.

End Answer Info . . . .