Huygens College Reflection

Assignment 3, Tuesday, Dec. 1, 2015

In the exercises, $=$ should be read as $=_{\mathsf{CL}}$.

1. Construct in each item a term P of CL with as few variables as possible such that:

$$\begin{aligned} P\,y &= y\,x && (1)\\ P\,x\,y &= y\,x && (2)\\ P\,x\,y &= y && (3)\\ P\,x &= x\,(x\,x) && (4)\\ P\,x\,y &= x\,x\,x && (5)\\ P\,x\,y &= x\,y\,y && (6)\\ P\,x\,y &= x\,(y\,y). && (7)\end{aligned}$$

2. Construct in each item a term Q of CL without variables such that:

$$\begin{aligned} Q &= Q\,\mathbf{K} && (8)\\ Q\,x &= x\,Q && (9)\\ Q &= Q\,\mathbf{I}\,\mathbf{I} && (10)\\ Q\,x &= Q\,x\,x. && (11)\end{aligned}$$

3.* If we add to CL the axiom $P = Q$, and we can then prove that $U = V$, we write

$$P = Q \vdash U = V.$$

Show that

(a) $\mathbf{I} = \mathbf{K} \vdash x = y$
$\mathbf{I} = \mathbf{S} \vdash x = y$
$\mathbf{K} = \mathbf{S} \vdash x = y.$

(b) We assume that $\nvdash x = y$, or in a different notation $x \neq_{\mathsf{CL}} y$. (This is true, but we don't prove it.) From this, we immediately get, using (a), that

$$\mathbf{I} \neq \mathbf{K},\ \mathbf{I} \neq \mathbf{S},\ \mathbf{K} \neq \mathbf{S}.$$

Prove that $\mathbf{KK} \neq \mathbf{K}$.

(c) Prove that there are no terms F, G (without variables) such that

$$F(x\,y) = x,\ G(x\,y) = y.$$

[Hint. From the outcome of a computation $P\,A$ one cannot see what the original argument and the function have been. For example, $0 \times 1 = 0 \times 2$ en $2^2 = \sqrt{16}$.]