This test consists of four exercises over 2 pages. It is advised to explain your approach and to check your answers carefully. You can score a maximum of 100 points. Each question indicates how many points it is worth. The test is closed book. You are NOT allowed to use a calculator, a computer or a mobile phone. You may answer in Dutch or in English. Please write clearly, and do not forget to put on each page: your name, your student number, and your werkcollege group. Put your student-card clearly visible at the corner of your table for inspection.

Notation Throughout the test, we denote for any alphabet $A$ and $a \in A$ by $|w|_a$ the number of $a$’s in the word $w \in A^*$, as it was introduced in the exercises.

1 Non-deterministic Finite Automata

a) Let $\mathcal{N}$ be the NFA given by the following diagram

\[ \begin{array}{c}
\mathcal{N} \\
q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_2 \\
q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_2 \\
\end{array} \]

i) Give a $\lambda$-NFA $\mathcal{N'}$ with one final state that accepts the same language as $\mathcal{N}$.

ii) Construct from $\mathcal{N'}$ a regular expression that generates the language accepted by $\mathcal{N'}$, using the procedure from the lecture. All intermediate steps belong to your answer.

b) Let $\mathcal{N}$ be the NFA over the alphabet $\{a, b\}$ be given by the following diagram.

\[ \begin{array}{c}
\mathcal{N} \\
q_0 \xrightarrow{a, b} q_1 \xrightarrow{b} q_3 \\
q_0 \xrightarrow{a} q_2 \\
\end{array} \]

Use the subset construction to obtain a DFA that accepts the same language as $\mathcal{N}$. Leave out unreachable states and clearly mark the states by the set of states they are generated from.
2 Pumping Lemma for Regular Languages

Let $A$ be the alphabet $\{a, b\}$ and $L$ the language

$$L = \{ vv^R \mid v \in A^*, |v|_a + |v|_b = 2k + 1, k \in \mathbb{N} \}.$$  

Use the pumping lemma to show that $L$ is not regular.  

Take care that the word, you choose in the contradiction, is indeed in $L$.  

(15pt)

3 Context Free Grammars

Let $A$ be the alphabet $\{a, b\}$ and $L$ again be the language $L = \{ vv^R \mid v \in A^*, |v|_a + |v|_b = 2k + 1, k \in \mathbb{N} \}$.

a) Give a grammar $G$ that generates the language $L$.  

b) Show that the word $abbbba$ is generated by $G$.  

c) Show that the words $aba$ and $abba$ are not generated.  

(15pt)

4 Push Down Automata

a) Let $\Sigma = \{a, b, c\}$ and the language $L$ be given by

$$L = \{ w \in \Sigma^* \mid |w|_a = |w|_b + |w|_c \}.$$  

Give a PDA with one state that accepts exactly the language $L$. Clearly indicate the stack alphabet you are using. Moreover, give the accepting computation for the word $abca$.  

(13pt)

b) Let $G$ be the grammar on the alphabet $\{a, b\}$ given by

$$S \to \lambda | aS | bSB$$
$$B \to b | aS$$

Construct a two state PDA that accepts the language generated by $G$.  

(12pt)