Talen en Automaten

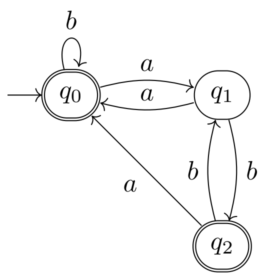
Test 2, Wed 28$^{\text{th}}$ Jan, 2015
13h45 – 15h45

This test consists of **four** exercises over **2 pages**. It is advised to explain your approach and to check your answers carefully. You can score a maximum of 100 points. Each question indicates how many points it is worth. The test is closed book. You are NOT allowed to use a calculator, a computer or a mobile phone. You may answer in Dutch or in English. Please write clearly, and do not forget to put on each page: your name, your student number, and your werkcollege group. Put your student-card clearly visible at the corner of your table for inspection.

Notation Throughout the test, we denote for any alphabet A and $a \in A$ by $|w|_a$ the number of a's in the word $w \in A^*$, as it was introduced in the exercises.

1 Non-deterministic Finite Automata

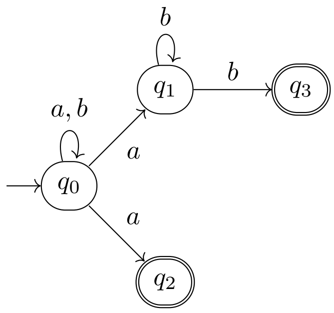
a) Let $\mathcal{N}$ be the NFA given by the following diagram



i) Give a λ-NFA $\mathcal{N}'$ with one final state that accepts the same language as $\mathcal{N}$. **(5pt)**

ii) Construct from $\mathcal{N}'$ a regular expression that generates the language accepted by $\mathcal{N}'$, using the procedure from the lecture. All intermediate steps belong to your answer. **(12pt)**

b) Let $\mathcal{N}$ be the NFA over the alphabet $\{a, b\}$ be given by the following diagram.



Use the subset construction to obtain a DFA that accepts the same language as $\mathcal{N}$. Leave out unreachable states and clearly mark the states by the set of states they are generated from. **(8pt)**

2 Pumping Lemma for Regular Languages

Let A be the alphabet $\{a, b\}$ and L the language

$$L = \{vv^R \mid v \in A^*, |v|_a + |v|_b = 2k + 1, k \in \mathbb{N}\}.$$

Use the pumping lemma to show that L is not regular. **(15pt)**
Take care that the word, you choose in the contradiction, is indeed in L.

3 Context Free Grammars

Let A be the alphabet $\{a, b\}$ and L again be the language

$$L = \{vv^R \mid v \in A^*, |v|_a + |v|_b = 2k + 1, k \in \mathbb{N}\}.$$

a) Give a grammar G that generates the language L. **(15pt)**

b) Show that the word *abbbba* is generated by G. **(5pt)**

c) Show that the words *aba* and *abba* are *not* generated. **(15pt)**

4 Push Down Automata

a) Let $\Sigma = \{a, b, c\}$ and the language L be given by

$$L = \{w \in \Sigma^* \mid |w|_a = |w|_b + |w|_c\}.$$

Give a PDA with one state that accepts exactly the language L. Clearly indicate the stack alphabet you are using. Moreover, give the accepting computation for the word *abca*. **(13pt)**

b) Let G be the grammar on the alphabet $\{a, b\}$ given by

$$S \to \lambda \mid aS \mid bSB$$
$$B \to b \mid aS$$

Construct a two state PDA that accepts the language generated by G. **(12pt)**