Talen en Automaten

Assignment 3, Tue 22nd Nov, 2016

Exercise teachers. The student groups are supervised by the following teachers:

Teacher	E-Mail	Room	Time
Michiel de Bondt	M.deBondt@math.ru.nl	HG00.058	10:45 – 12:30
Bas Steeg	steegbas@gmail.com	HG02.032	10:45 – 12:30
Demian Janssen	wd.janssen@student.ru.nl	HG01.058	10:45 – 12:30
Jan Martens	jan-martens@hotmail.com	HG01.139	10:45 – 12:30
Ties Robroek	ties.robroek@student.ru.nl	HG02.028	10:45 – 12:30
Sjoerd Hemels	sjoerd0707@live.nl	HG03.632	10:45 – 12:30
Rick Erkens	rjarickerkens@gmail.com	HG02.028	**13:45 – 15:30**

Postboxes are located in the Mercator building on the ground floor. There will be 7 boxes labelled with *Talen en Automaten* and the corresponding group teacher's name. There will be 1 box, the *Uitleverbak*, for work that hasn't been picked up at the exercise hours.

Handing in your answers: There are two options:

1. E-mail: Send your solutions by e-mail to your exercise class teacher (see above) with subject "**T&A: assignment 3**". This e-mail should only contain a single PDF document as attachment (unless explicitly stated otherwise). Before sending an e-mail make sure:
   - the file is a PDF document
   - your name is part of the filename (for example MyName_assignment-3.pdf)
   - your name and student number are included in the document (they will be printed).
2. Post box: Put your solutions in the appropriate post box (see above). Before putting your solutions in the post box make sure:
   - your name, student number, and IC, KI or Wiskunde are written clearly on the document.

Deadline: Tue 29th Nov, 2016, 13:45 (in Nijmegen!)

Goals: After completing these exercises successfully you should be able to construct an NFA from a language description, to construct an NFA-λ from a regular expression, and to determinise an NFA-λ.

There are 3 mandatory exercises, worth **10 points** in total. There are 2 more, extra hard, exercises. Be aware that these exercises are just for fun, you cannot earn any points with them.

1 NFAs and Their Languages

a) Let $A = \{0, 1, 2\}$ and let L be the set of words in which the last digit occurs twice with no larger digit in between:

$$L = \{w \in A^* \mid \exists x \in A.\ \exists u, v \in A^*.\ w = uxvx, \text{ and there is no } y \text{ in } v \text{ s.t. } x < y\},$$

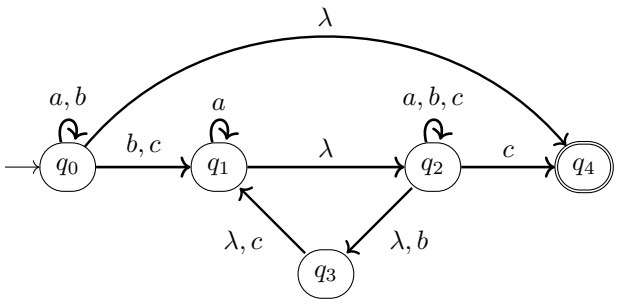
where $0 < 1 < 2$. For example, $00, 2111, 2102 \in L$, but $1, 121 \notin L$.

Construct an NFA that accepts L, and show that 2101 is accepted, but 121 not. **(4pt)**

b) Show that for every DFA $D = (Q, \delta, q_0, F)$ over a finite alphabet, there is an NFA $N = (Q', \delta', q_0', F')$ on the same alphabet that accepts the same language. This requires a proof that $\mathcal{L}(q_0) = \mathcal{L}(q_0')$. **(2pt)**

2 Determinising NFAs-λ

Let M be the NFA-λ given by the following graph.



Use the powerset construction from the lecture to turn M into a DFA M' that accepts the same language. Leave out unreachable states, and clearly indicate how a state of M' corresponds to a subset of states of M. **(2pt)**

3 From Regular Expressions to NFAs-λ

Let e be the regular expression $(b + a^*)a$.

Use the "toolkit" from the lecture to construct an NFA-λ that accepts $\mathcal{L}(e)$. The (non-trivial) intermediate steps must be given as part of the solution. **(2pt)**

4 Fun Exercises – Properties of Regular Languages

a) Using that regular languages are closed under complement and intersection, show that for regular languages L_1, L_2 their difference $L_1 \setminus L_2$ is regular as well.

b) Give an algorithm that decides whether for a regular expression e its language $\mathcal{L}(e)$ is empty.

c) Give an algorithm that checks for given regular expressions e_1, e_2 whether their languages are equal: $\mathcal{L}(e_1) = \mathcal{L}(e_2)$.

5 Fun Exercise – Constructing an NFA-λ

Give an NFA-λ over $A = \{a\}$ such that it rejects some string and the length of the shortest rejected string is strictly greater than the number of states.