

Talen en Automata

Assignment 5, Tuesday 6th December, 2016

Exercise teachers. The student groups are supervised by the following teachers:

Teacher	E-Mail	Room	Time
Michiel de Bondt	M.deBondt@math.ru.nl	HG00.058	10:45 – 12:30
Bas Steeg	steegbas@gmail.com	HG02.032	10:45 – 12:30
Demian Janssen	wd.janssen@student.ru.nl	HG01.058	10:45 – 12:30
Jan Martens	jan-martens@hotmail.com	HG01.139	10:45 – 12:30
Ties Robroek	ties.robroek@student.ru.nl	HG02.028	10:45 – 12:30
Sjoerd Hemels	sjoerd0707@live.nl	HG03.632	10:45 – 12:30
Rick Erkens	rjarickerkens@gmail.com	HG02.028	13:45 – 15:30

Postboxes are located in the Mercator building on the ground floor. There are 7 boxes labelled with *Talen en Automaten* and the corresponding group teacher's name. There is 1 box, the *Uitleverbak*, for work that hasn't been picked up at the exercise hours.

Handing in your answers: There are two options:

1. E-mail: Send your solutions by e-mail to your exercise class teacher (see above) with subject “**T&A: assignment 5**”. This e-mail should only contain a single PDF document as attachment (unless explicitly stated otherwise). Before sending an e-mail make sure:
 - the file is a PDF document
 - your name is part of the filename (for example MyName_assignment-5.pdf)
 - your name and student number are included in the document (they will be printed).
2. Post box: Put your solutions in the appropriate post box (see above). Before putting your solutions in the post box make sure:
 - your name, student number, and IC, KI or Wiskunde are written clearly on the document.

Deadline: Tuesday 13th December, 2016, 13:45 (in Nijmegen!)

Goals: After completing these exercises successfully you should be able to read context-free grammars, as well as write down grammars for context-free languages and regular languages. The total number of points is 10.

There are 3 mandatory exercises, worth **10 points** in total. There are 3 more, extra hard, exercises. Be aware that these exercises are just for fun, you cannot earn any points with them.

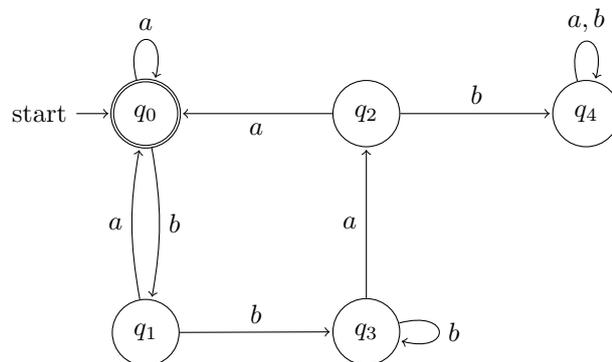
1. Let $\Sigma = \{a, b\}$. Consider the following context-free grammar

$$G_1 = \boxed{S \rightarrow aS \mid Sb \mid ab \mid SS}$$

- (a) Show that the grammar is ambiguous by giving two left-most (1pt) derivations of $abbb$.

- (b) Give a regular expression for $L_1 := \mathcal{L}(G_1)$. (1pt)
- (c) Give a non-ambiguous regular grammar for L_1 . (1pt)
2. For each of the following languages construct a context-free grammar that generates the language, and explain why your answer is correct. (In case the language is regular, try to give a regular grammar.)
- (2 points) $L_2 = \{a^n b^{n+m} a^m \mid n, m \geq 0\}$
- (2 points) $L_3 = \{w \in \{a, b\}^* \mid |w|_a \text{ is even}\}$
- (2 points) $L_4 = \mathcal{L}((aa^*b)^*a(a+bb)^*)$

3. (1 point) Consider the following DFA M ,



Construct a context-free grammar that generates $\mathcal{L}(M)$.

Fun Exercises

1. Construct a context-free grammar that generates the following language:

$$L_5 = \{w \in \{a, b\}^* \mid |w| = 2k + 1 \text{ and } w_1 = w_{k+1}\},$$

where w_i denotes the i -th symbol in a word w . That is, L_5 consists of all words of odd length that have the same symbol in the first and middle positions.

2. $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, \times, +, (,)\}$. Construct a context-free grammar that generates the language

$$L_6 = \{w \in \Sigma^* \mid w \text{ is a well-formed arithmetical expressions}\}$$

NB. $2 + 3 + 4 \times 5$ and $((2 + 3) + 4) \times 5$ and $((2 + 3)) + 4 \times 5$ are well-formed. $2 + (3 + 4 \times 5$ and $(2 + 3) + 4) \times 5$ and $) ($ are not.

3. Suppose that L and L' are context-free languages. Show that both L^* and $L L'$ are context-free languages.