Talen en Automata

Assignment 6, Tuesday 20th December, 2016

Exercise teachers. The student groups are supervised by the following teachers:

Teacher	E-Mail	Room	Time
Michiel de Bondt	M.deBondt@math.ru.nl	HG00.058	10:45 – 12:30
Bas Steeg	steegbas@gmail.com	HG02.032	10:45 – 12:30
Demian Janssen	wd.janssen@student.ru.nl	HG01.058	10:45 – 12:30
Jan Martens	jan-martens@hotmail.com	HG01.139	10:45 – 12:30
Ties Robroek	ties.robroek@student.ru.nl	HG02.028	10:45 – 12:30
Sjoerd Hemels	sjoerd0707@live.nl	HG03.632	10:45 – 12:30
Rick Erkens	rjarickerkens@gmail.com	HG02.028	**13:45 – 15:30**

Postboxes are located in the Mercator building on the ground floor. There are 7 boxes labelled with *Talen en Automaten* and the corresponding group teacher's name. There is 1 box, the *Uitleverbak*, for work that hasn't been picked up at the exercise hours.

Handing in your answers: There are two options:

1. E-mail: Send your solutions by e-mail to your exercise class teacher (see above) with subject "**T&A: assignment 6**". This e-mail should only contain a single PDF document as attachment (unless explicitly stated otherwise). Before sending an e-mail make sure:
   - the file is a PDF document
   - your name is part of the filename (for example MyName_assignment-6.pdf)
   - your name and student number are included in the document (they will be printed).
2. Post box: Put your solutions in the appropriate post box (see above). Before putting your solutions in the post box make sure:
   - your name, student number, and IC, KI or Wiskunde are written clearly on the document.

Deadline: Tuesday 10th January, 2017, 13:45 (in Nijmegen!)

Goals: After completing these exercises successfully you should be able to show that a language is context free by giving a push down automaton (PDA) that accepts it. Moreover, you should be able to turn a context free grammar into a PDA and vice versa.

There are 2 mandatory exercises, worth **10 points** in total. There are 2 more, extra hard, exercises. Be aware that these exercises are just for fun, you cannot earn any points with them.

1 Push Down Automata

Let A be the alphabet $\{a, b\}$.

a) Recall the language L of palindromes:

$$L = \{w \in A^* \mid w = w^R\}.$$

Give a PDA that accepts L, and show that aba is accepted but ab is not. **(2pt)**

b) Let L be given by

$$L = \{w \in A^* \mid |w|_a = |w|_b\}.$$

Construct a PDA that accepts L, and show that the word $abba$ is accepted but aab is not. **(3pt)**

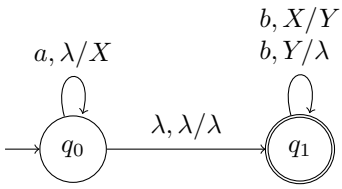
2 CFGs and PDAs

a) The following grammar generates expressions such as $(a + a) + a$.

$$S \longrightarrow F \mid S + S$$
$$F \longrightarrow a \mid (S)$$

Use the construction given in the lecture to give a two state PDA that accepts the language generated by this grammar. **(2pt)**

b) Let M be the PDA over the alphabet $\{a, b\}$ and with stack alphabet $\{X, Y\}$ given by the following graph.

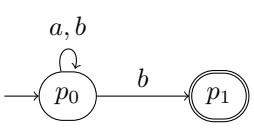


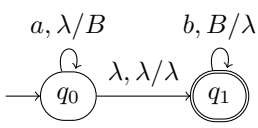
Use the procedure given in the lecture to construct a grammar that generates $\mathcal{L}(M)$, and give the derivation for the word abb. **(3pt)**

3 Fun Exercises – Closure Properties of CFLs

a) Let $\mathcal{A} = (Q_1, q_{1,0}, F_1, \delta_1 : Q_1 \times \Sigma \to \mathcal{P}(Q_1))$ be an NFA accepting the regular language L and let $\mathcal{B} = (Q_2, \Sigma, \Gamma, q_{2,0}, F_2, \delta_2 : Q_2 \times \Sigma_\lambda \times \Gamma_\lambda \to \mathcal{P}(Q_2 \times \Gamma_\lambda))$ be a PDA that accepts the context free language D. Show that $L \cap D$ is context free by defining a PDA accepting it. Use $Q_1 \times Q_2$ as state space.

b) Let $\mathcal{A}$ and $\mathcal{B}$ be given as in the following diagrams.





Apply your construction from **????** to these automata to define a PDA that accepts $\mathcal{L}(\mathcal{A}) \cap \mathcal{L}(\mathcal{B})$.

4 Fun Exercises – Beyond CFLs

We extend PDAs to two-stack PDAs (PDA_2). A PDA_2 M is given by a tuple $(Q, \Sigma, \Gamma, \delta, q_0, F)$ just like a PDA, except that

$$\delta : Q \times \Sigma_\lambda \times \Gamma_\lambda \times \Gamma_\lambda \to \mathcal{P}(Q \times \Gamma_\lambda \times \Gamma_\lambda).$$

Give a suitable extension of the acceptance condition for PDAs, and show that two-stack PDAs are computationally more powerful than PDAs, by showing that there is a PDA_2 that accepts

$$\{a^n b^n c^n \mid n \in \mathbb{N}\}.$$