Talen en Automaten

Test 1, Mon 7th Dec, 2015
15h45 – 17h30

This test consists of **four** exercises over **7 pages**. Explain your approach, and **write your answer to each exercise on a separate page**. You can score a maximum of 100 points, and each question indicates how many points it is worth. The test is closed book. You are NOT allowed to use a calculator, a computer or a mobile phone. You may answer in Dutch or in English. Please write clearly, and do not forget to put on each page: your name and your student number.

Notation Throughout the test, we denote for any alphabet A, $w \in A^*$ and $a \in A$ by $|w|_a$ the number of a's in w, as it was introduced in the lecture. Moreover, recall that v is a *subword* of w if $w = xvy$ for some words x, y.

1 Induction

Let A and B be finite alphabets and $f : A \to B^*$ a map from A to words over B.

a) Define by induction a map $\overline{f} : A^* \to B^*$ that replaces in a word $w \in A^*$ all letters a by $f(a)$. **(5pt)**

b) Let $A = \{a, b\}$ and $f : A \to A^*$ be given by $f(a) = b$ and $f(b) = abb$.

i) Give a word $w \in A^*$ such that $\overline{f}(w) = babbbb$. **(5pt)**

ii) Show by induction that $\left|\overline{f}(w)\right|_b = |w|_a + 2|w|_b$. **(10pt)**

2 Regular Languages [Write your answers on a separate page]

a) Let $A = \{a, b\}$ and

$$L_1 = \{w \in A^* \mid |w|_a \text{ is even}\}$$
$$L_2 = \mathcal{L}\left((a+b)^* abba (a+b)^*\right)$$
$$L_3 = \{w \in A^* \mid w \text{ does not contain the subword } abb\}.$$

Explain for each $i = 1, 2, 3$ whether and why ${L_i}^* = L_i$. **(10pt)**

b) Let $A = \{a, b\}$ and

$$L = \{w \in A^* \mid aba \text{ occurs twice as subword in } w\}.$$

Give a regular expression e, such that $\mathcal{L}(e) = L$. Explain your answer. **(10pt)**

3 Deterministic Finite Automata [Write your answers on a separate page]

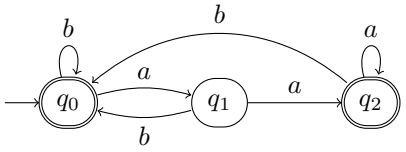
a) Let $A = \{a, b, c\}$ and let

$$L = \{w \in A^* \mid ab \text{ occurs an odd number of times as subword in } w\}.$$

i) Give a DFA M with $\mathcal{L}(M) = L$. Explain your answer. **(10pt)**

ii) Show that *caba* is accepted, and that *abbab* is not accepted. **(5pt)**

b) Let $A = \{a, b\}$ and the DFA M over A be given by



Use the procedure from the lecture to construct a regular expression e with $\mathcal{L}(e) = \mathcal{L}(M)$. **(10pt)**

4 Non-Deterministic Finite Automata **[Write your answers on a separate page]**

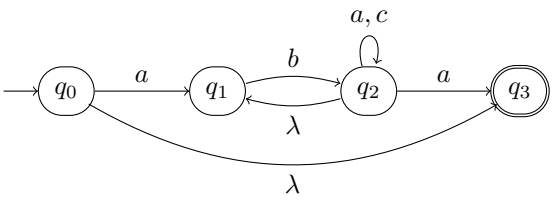
a) Let $A = \{0, 1, 2\}$ and let L be the language of words in which the digits occur only in increasing order, i.e.,

$$L = \{x_1 \cdots x_n \mid n \in \mathbb{N}, \forall i.\ x_i \in A \text{ and } \forall i \leq j.\ x_i \leq x_j\}.$$

i) Show that L is regular by constructing an NFA-λ that accepts L. **(10pt)**

ii) Show that your automaton accepts 002 and rejects 21. **(5pt)**

b) Let $A = \{a, b, c\}$ and the NFA-λ M over A be given by



Use the procedure from the lecture to construct a DFA D with $\mathcal{L}(D) = \mathcal{L}(M)$. Indicate clearly from which subset of states of M a state in D originates. **(10pt)**

c) Let e be the regular expression $a + (b + 1)^*$.

Use the procedure from the lecture to construct an NFA-λ M with $\mathcal{L}(M) = \mathcal{L}(e)$. **(10pt)**