Talen en Automaten

Test 2, Thu 21st Jan, 2016

This test consists of **four** exercises over **6 pages**. Explain your approach. You can score a maximum of 100 points, and each question indicates how many points it is worth. The test is closed book. You are NOT allowed to use a calculator, a computer or a mobile phone. You may answer in Dutch or in English. Please write clearly, and do not forget to put on each page: your name and your student number.

Notation Throughout the test, we denote for any alphabet A and $a \in A$ by $|w|_a$ the number of a's in the word $w \in A^*$, as it was introduced in the lecture.

Write the answer to each exercise on a separate sheet!

1 Non-Regular Languages

Write your answers on a separate sheet

Let $A = \{a, b\}$.

a) We define the language L to be

$$L = \{wb^n \mid w \in A^*, |w| = n\}.$$

Show that L is not regular. **(5pt)**

b) Show that the language $L = \{w \in A^* \mid |w|_a = |w|_b\}$ is not regular, using the Pumping Lemma. **(10pt)**

2 Context Free Grammars

Write your answers on a separate sheet

Fix $A = \{a, b\}$ for this exercise.

a) Let L be the language over A given by $L = \{a^n b^k a^m \mid k = n + m\}$.

- **i)** Construct a CFG G such that $\mathcal{L}(G) = L$. **(10pt)**
- **ii)** Give a derivation for the word $aabbba \in L$. **(5pt)**
- **iii)** Show that the word aba is not generated. **(5pt)**

b) Let G be the following CFG over A.

$$S \longrightarrow US \mid \lambda$$
$$U \longrightarrow aa \mid ab \mid bb \mid ba$$

- **i)** Give a precise description of $\mathcal{L}(G)$ using set notation. **(5pt)**
- **ii)** Is $\mathcal{L}(G)$ a regular language? Explain your answer by either giving a reason why it is not or by giving a regular grammar for $\mathcal{L}(G)$. **(10pt)**

3 Push Down Automata I

Write your answers on a separate sheet

Let M be the PDA with

$$Q = \{q_0, q_1, q_2\} \qquad \delta(q_0, b, \lambda) = \{\langle q_1, B\rangle\}$$
$$\Sigma = \{a, b, c\} \qquad \delta(q_0, b, C) = \{\langle q_1, \lambda\rangle\}$$
$$\Gamma = \{B, C\} \qquad \delta(q_0, c, \lambda) = \{\langle q_2, C\rangle\}$$
$$F = \{q_0\} \qquad \delta(q_0, c, B) = \{\langle q_2, \lambda\rangle\}$$
$$\delta(q_1, a, \lambda) = \{\langle q_0, \lambda\rangle\}$$
$$\delta(q_2, a, \lambda) = \{\langle q_0, \lambda\rangle\}$$

a) Draw a state diagram for M. **(5pt)**

b) Check which of the following words is in $\mathcal{L}(M)$ and explain your answer: *abcb* and *baca*. **(5pt)**

c) Is $\mathcal{L}\left((ca)^*(ba)^*\right) \subseteq \mathcal{L}(M)$? Explain your answer. **(5pt)**

d) Give a precise description of $\mathcal{L}(M)$ using set notation. **(5pt)**

4 Push Down Automata II

Write your answers on a separate sheet

a) **i)** Let $A = \{a, b\}$ and let L be the language $L = \{w \in A^* \mid |w|_a = 2|w|_b + 1\}$.
Show that L is context free by giving a PDA that accepts it. **(10pt)**

ii) Show that *aaba* and *baaa* are accepted, by giving the accepting computations. **(5pt)**

iii) Show that *aab* is not accepted by your PDA. **(5pt)**

b) Let G be the grammar on the alphabet $\{a, b\}$ given as follows.

$$S \to \lambda \mid aX \mid bY$$
$$X \to bYb \mid bb$$
$$Y \to aXa \mid aa$$

Construct a PDA that accepts $\mathcal{L}(G)$, using the procedure given in the lecture. **(10pt)**