

Regular Languages & Finite Automata

Now we are going to play a different 'ball-game'. Let $\Sigma = \{a, b\}$

S	\rightarrow	\emptyset	$ $	E
E	\rightarrow	λ	$ $	aEa $ $ bEb
\emptyset	\rightarrow	a	$ $	b $ $ $a\emptyset a$ $ $ $b\emptyset b$

G_1 a *context-free grammar*

Productions, always start with S

$$S \Rightarrow E \Rightarrow aEa \Rightarrow abEba \Rightarrow abba$$

$$S \Rightarrow E \Rightarrow bEb \Rightarrow baEab \Rightarrow babEbab \Rightarrow babaEabab \Rightarrow babaabab$$

$$S \Rightarrow \emptyset \Rightarrow b\emptyset b \Rightarrow bab$$

$$S \Rightarrow \emptyset \Rightarrow b\emptyset b \Rightarrow ba\emptyset ab \Rightarrow babab$$

$$L(G) = \{w \in \Sigma^* \mid w \text{ is a palindrome}\},$$

where w is a palindrome if $w = w^R$

and w^R denotes word reversal. (See earlier lectures.)

Let Σ be a finite alphabet

A *context-free grammar* G over Σ needs a finite set V of *auxiliary* (help) symbols and consists of productionrules of the form

$$X \rightarrow w$$

with $X \in V$ and $w \in (\Sigma \cup V)^*$. There is an $S \in V$ (*start*)

Using G a language is generated using a relation \Rightarrow ('*produces*') defined as follows

(where u, v, w, x, y are arbitrary elements of $(\Sigma \cup V)^*$)

$$\begin{aligned} X \rightarrow w & \text{ implies } xXy \Rightarrow xwy \\ u \Rightarrow v, v \Rightarrow w & \text{ implies } u \Rightarrow w \end{aligned}$$

The *language generated by* G is

$$L(G) = \{w \in \Sigma^* \mid S \Rightarrow w\}$$

Notation. $X \rightarrow w_1 \mid w_2$ abbreviates $\begin{array}{l} X \rightarrow w_1 \\ X \rightarrow w_2 \end{array}$

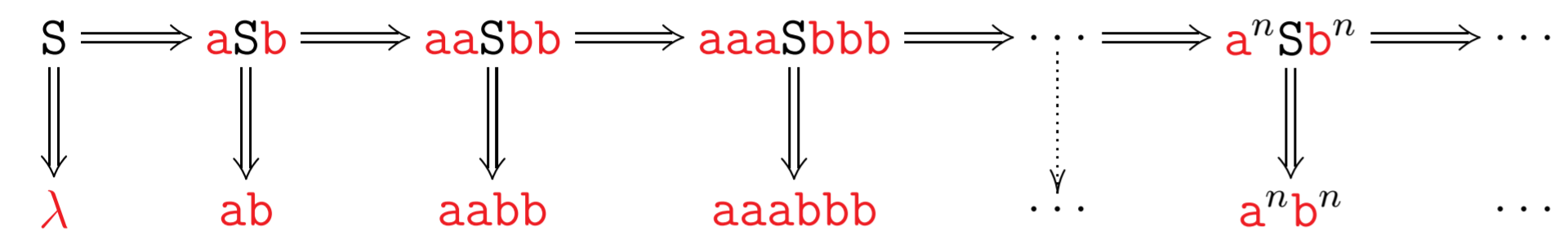
A language L is *context-free* if $L = L(G)$ for some context-free G

Is there a grammar G_2 such that

$$L(G_2) = \{a^n b^n \mid n \geq 0\}?$$

$$\boxed{S \rightarrow \lambda \mid aSb} \quad G_2$$

All possible productions



What is a grammar G_3 such that

$$L(G_3) = \{a^n b^n \mid n > 0\}?$$

$$\boxed{S \rightarrow ab \mid aSb} \quad G_3$$

Let $\Sigma = \{a, b, c\}$

Claim. $L = \{a^n b^m c^{2n+1} \mid n \geq 0\}$ is context-free

Let us first show

$L' = \{a^n c^{2n+1} \mid n \geq 0\}$ is context-free

Use G' given by $S \rightarrow c \mid aScc$

For L use G given by $S \rightarrow Bc \mid aScc$
 $B \rightarrow \lambda \mid bB$

Fact. $\{a^n b^n c^n \mid n \geq 0\}$ is *not* context-free

Let $\Sigma = \{a, b\}$

Claim. $L = \{w \mid w \text{ ends with } bb\}$ is context-free

Use G given by

S	\rightarrow	A	b	b
A	\rightarrow	A	a	$ $
		A	b	$ $
		λ		

Let $\Sigma = \{a, b\}$

Claim. $L = \{w \mid w \text{ contains exactly two } a\text{'s}\}$ is context-free

Use G given by

S	\rightarrow	A	A	
A	\rightarrow	B	a	B
B	\rightarrow	B	b	$ $
		λ		