Regular Languages & Finite Automata

Now we are going to play a different 'ball-game'. Let $\Sigma = \{a, b\}$

S	$\longrightarrow$	O | E
E	$\longrightarrow$	λ | aEa | bEb
O	$\longrightarrow$	a | b | aOa | bOb

G_1 a *context-free grammar*

Productions, always start with S

S $\Rightarrow$ E $\Rightarrow$ aEa $\Rightarrow$ abEba $\Rightarrow$ abba

S $\Rightarrow$ E $\Rightarrow$ bEb $\Rightarrow$ baEab $\Rightarrow$ babEbab $\Rightarrow$ babaEabab $\Rightarrow$ babaabab

S $\Rightarrow$ O $\Rightarrow$ bOb $\Rightarrow$ bab

S $\Rightarrow$ O $\Rightarrow$ bOb $\Rightarrow$ baOab $\Rightarrow$ babab

$L(G) = \{w \in \Sigma^* \mid w$ is a palindrome$\}$,

where w is a palindrome if $w = w^R$

and w^R denotes word reversal. (See earlier lectures.)

Let Σ be a finite alphabet

A *context-free grammar* G over Σ needs a finite set V of *auxiliary* (help) symbols

and consists of productionrules of the vorm

$$X \to w$$

with $X \in V$ and $w \in (\Sigma \cup V)^*$. There is an $S \in V$ (*start*)

Using G a language is generated using a relation $\Rightarrow$ ('*produces*') defined as follows

(where u, v, w, x, y are arbitrary elements of $(\Sigma \cup V)^*$)

$$\begin{array}{rcl} X \to w & \text{implies} & xXy \Rightarrow xwy \\ u \Rightarrow v,\ v \Rightarrow w & \text{implies} & u \Rightarrow w \end{array}$$

The *language generated by G* is

$$L(G) = \{w \in \Sigma^* \mid S \Rightarrow w\}$$

Notation. $X \to w_1 \mid w_2$ abbreviates $\begin{array}{lcl} X & \to & w_1 \\ X & \to & w_2 \end{array}$

A language L is *context-free* if $L = L(G)$ for some context-free G

Is there a grammar G_2 such that

$L(G_2) = \{a^n b^n \mid n \geq 0\}$?

S	$\rightarrow$	λ | aSb

G_2

All possible productions

$$S \Longrightarrow aSb \Longrightarrow aaSbb \Longrightarrow aaaSbbb \Longrightarrow \cdots \Longrightarrow a^n S b^n \Longrightarrow \cdots$$

$$\Downarrow \qquad \Downarrow \qquad \Downarrow \qquad \Downarrow \qquad \vdots \qquad \Downarrow$$

$$\lambda \qquad ab \qquad aabb \qquad aaabbb \qquad \cdots \qquad a^n b^n \qquad \cdots$$

What is a grammar G_3 such that

$L(G_3) = \{a^n b^n \mid n > 0\}$?

S	$\rightarrow$	ab| aSb

G_3

Let $\Sigma = \{a, b, c\}$

Claim. $L = \{a^n b^m c^{2n+1} \mid n \geq 0\}$ is context-free

Let us first show

$L' = \{a^n c^{2n+1} \mid n \geq 0\}$ is context-free

Use G' given by

S	$\rightarrow$	c | aScc

For L use G given by

S	$\rightarrow$	Bc | aScc
B	$\rightarrow$	λ | bB

Fact. $\{a^n b^n c^n \mid n \geq 0\}$ is *not* context-free

Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$

Claim. $L = \{w \mid w$ ends with $\mathtt{bb}\}$ is context-free

Use G given by

$$\begin{array}{lcl} \mathtt{S} & \to & \mathtt{Abb} \\ \mathtt{A} & \to & \mathtt{Aa} \mid \mathtt{Ab} \mid \lambda \end{array}$$

Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$

Claim. $L = \{w \mid w$ contains exactly two $\mathtt{a}$'s$\}$ is context-free

Use G given by

$$\begin{array}{lcl} \mathtt{S} & \to & \mathtt{AA} \\ \mathtt{A} & \to & \mathtt{BaB} \\ \mathtt{B} & \to & \mathtt{Bb} \mid \lambda \end{array}$$