Final lecture: Formal Languages, Grammars and Automata

- Pushdown Automata and Context Free Grammars (recall)
- Where does this topic go from here . . . ?
- Typical exam exercises: see the exercises of the "werkcollege"; we provide some additional ideas of exercises.

A pushdown automaton is a sextuple $\langle Q, \Sigma, \Gamma, \delta, q_0, F\rangle$ with

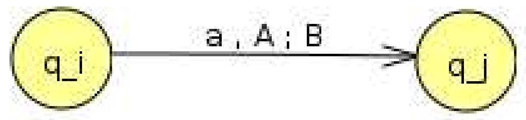
Q	a finite set of states
q_0	an element of Q, the initial state
F	a subset of Q
Σ	a finite set of symbols (input alphabet)
Γ	the *stack alphabet*
δ	a map ('afbeelding')
	$\delta : Q \times (\Sigma \cup \{\lambda\}) \times (\Gamma \cup \{\lambda\}) \to \mathcal{P}(Q \times \Gamma \cup \{\lambda\})$

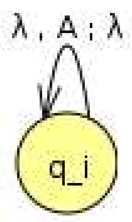
We write e.g. $\delta(q_i, a, A) = \{[q_j, B], [q_k, C]\}$

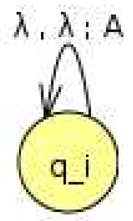
We understand the $Q, \Sigma, \mathcal{P}, \lambda$. New is the Γ: alphabet of stack symbols

The stack is not mentioned, but it is used in the operation of the PDA!

$[q', B] \in \delta(q, a, A)$: you can read a, pop A and push B







$$L(M) = \{w \in \Sigma^* \mid [q_0, w, \lambda] \vdash^* [q_i, \lambda, \lambda] \;\&\; q_i \in F\}$$

- Study of languages. Natural languages and computer languages.

  When are two languages the same? How do you parse or compile a language? In what complexity class is a language?

- Extending the stack to a tape that one can navigate over: Turing Machine, universal model of computation.

  See the course IBC003: Berekenbaarheid.

- Automata and grammars as small computing devices: simple, but well-understood. Slogan:

  The basic building blocks are simple;

  Complexity is an emerging property.

  E.g. genes and phenotype, Conway's game of life, Lindenmayer-systems (see next hour).

Typical exercises 1: Transformations (regular)

Let $L = \{w \in \{a,b\}^* \mid \#_a(w) \text{ is even}\}$

(i) Construct a regular expression e such that $L(e) = L$

(ii) Construct a DFA M such that $L(M) = L$

(iii) Construct a regular grammar G such that $L(G) = L$

Typical exercises 2: Transformations (context-free)

Let $L = \{a^n b^m \mid n > m\}$

(i) Construct a PDA M such that $L(M) = L$

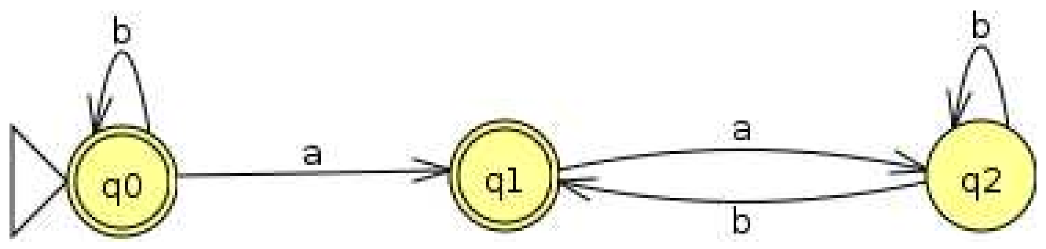
(ii) Construct a grammar G such that $L(G) = L$

(iii) Show that L is not regular.

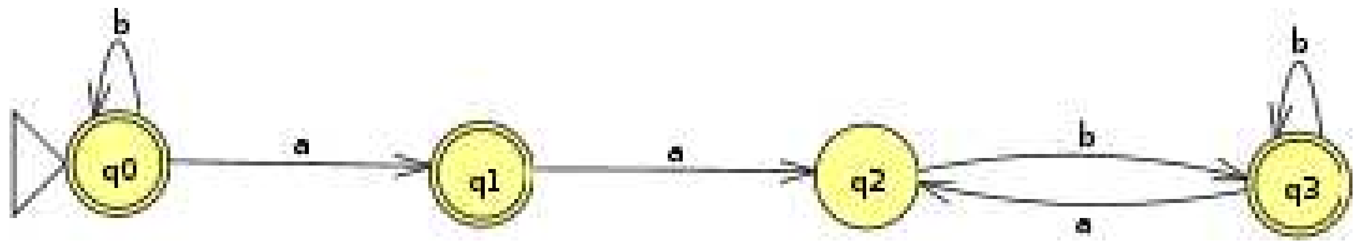
Typical exercises 3: Equivalences

(i) Let $e_1 = u^*(u \cup v)^*$ and $e_2 = (u \cup vu^*)^*$, where $u, v \in \Sigma^*$

Show that $L(e_1) = L(e_2)$

(ii) Let $M_1 =$ 

and

$M_2 =$ 

Show that $L(M_1) = L(M_2)$

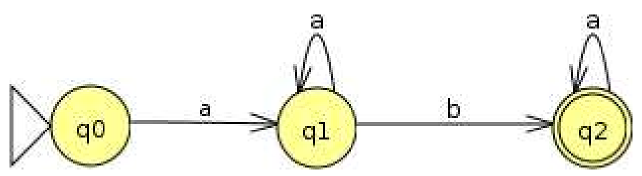
Typical exercises 4: the 'square'

Given is $\Sigma = \{a, b\}$

Consider

$$L = \{a^n b a^m \mid n > 0,\ m \geq 0\}$$

$$e = aa^*ba^*$$

$M =$ 

$$G = \boxed{\begin{array}{lcl} S & \rightarrow & aA \\ A & \rightarrow & aA \mid bB \\ B & \rightarrow & aB \mid \lambda \end{array}}$$

Show that $L = L(e) = L(G) = L(M)$.

[Hint. Showing $L \subseteq L(e) \subseteq L(G) \subseteq L(M) \subseteq L$ suffices!]

(iii) Let $G_1 = \begin{array}{rcl} S & \to & AB \\ A & \to & aA \mid \lambda \\ B & \to & bB \mid \lambda \end{array}$ and $G_2 = \begin{array}{rcl} S & \to & AB \mid A \mid B \mid \lambda \\ A & \to & aA \mid a \\ B & \to & bB \mid b \end{array}$

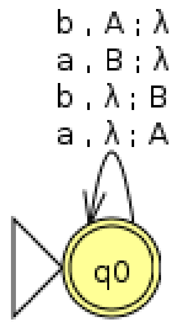
Show that $L(G_1) = L(G_2)$

Typical exercises 6 (*): the 'triangle'

Given is $\Sigma = \{a, b\}$

Consider

$$L = \{w \in \Sigma^* \mid \#_a(w) = \#_b(w)\}$$

$M =$ 

$$G = \begin{array}{|lcl|} \hline S & \to & aA \mid bB \mid \lambda \\ A & \to & bS \mid aAA \\ B & \to & aS \mid bBB \\ \hline \end{array}$$

(i) Show that $L = L(M)$

(ii) Show that $L = L(G)$