Exercises lecture 1
Formal languages, grammars, and automata
April 19, 2013

1. Regular languages

Read: Chapter 1 and Section 2.1 of the Reader Ruohonen; the slides of the course on the webpage.

Exercise 2 can be handed in with Nico Broeder or Jasper Derikx.

NB: Exercise 4 is only for devotees, to show that the subject is non-trivial.

For a \( w \in \Sigma^* \), let \( \#(w) \) be the number of symbols in \( w \); moreover for \( s \in \Sigma \), let \( \#_s(w) \) be the number of occurrences of \( s \) in \( w \). For example \( \#(aab) = 3 \), \( \#_a(aab) = 2 \), and \( \#_b(aab) = 1 \).

1. What are \( L_1 = L((a \cup b)^*) \), \( L_2 = L((a^* b^*)^*) \), and \( L_3 = ((ab^*)^*) \). Show that precisely two of these languages are equal.

2. (a) Give a regular expression for \( \{ w \in \{a,b,c\}^* \mid \#(w) = 3 \} \).

   (b) Same for \( \{ w \in \{a,b,c\}^* \mid \#(w) \geq 3 \} \).

   (c) Same for \( \{ w \in \{a,b\}^* \mid aa \) occurs exactly twice in \( w \} \).

   [Hint. Beware of the string \( aaa \)!]

3. Prove that

   \( \{ w \in \{a,b\}^* \mid bb \) does not occur in \( w \} = L(a^*(baa^*)^*b\lambda) \),

   where \( b\lambda = (b \cup \lambda) \). We have omitted some parentheses; the full regular expression is \((a^*)(((ba)(a^*))^*(b \cup \lambda))\).

4. [This exercise is at the moment rather hard, later less so!]

   Show that the language

   \( \{ w \in \{a,b\}^* \mid \#_a(w) \) and \( \#_b(w) \) are even \}

   is regular.

   Easier is to show this for

   \( \{ w \in \{a,b\}^* \mid \#_a(w) \) is even \}. 
