Exercises lecture 1
Formal languages, grammars, and automata

April 19, 2013

1. Regular languages

Read: Chapter 1 and Section 2.1 of the Reader Ruohonen; the slides of the course on the webpage.
Exercise 2 can be handed in with Nico Broeder or Jasper Derikx.
NB: Exercise 4 is only for devotees, to show that the subject is non-trivial.

For a $w \in \Sigma^*$, let $\#(w)$ be the number of symbols in w; moreover for $s \in \Sigma$, let $\#_s(w)$ be the number of occurrences of s in w. For example $\#(aab) = 3$, $\#_a(aab) = 2$, and $\#_b(aab) = 1$.

1. What are $L_1 = L((a \cup b)^*)$, $L_2 = L((a^*b^*)^*)$, and $L_3 = ((ab^*)^*)$. Show that precisely two of these languages are equal.
2. (a) Give a regular expression for
$$\{w \in \{a, b, c\}^* \mid \#(w) = 3\}.$$
(b) Same for
$$\{w \in \{a, b, c\}^* \mid \#(w) \geq 3\}.$$
(c) Same for
$$\{w \in \{a, b\}^* \mid aa \text{ occurs exactly twice in } w\}.$$
[Hint. Beware of the string aaa!]
3. Prove that
$$\{w \in \{a, b\}^* \mid bb \text{ does not occur in } w\} = L(a^*(baa^*)^*b?),$$
where $b? = (b \cup \lambda)$. We have omitted some parentheses; the full regular expression is $((a^*)((((ba)(a^*))^*)(b \cup \lambda)))$.
4. [This exercise is at the moment rather hard, later less so!]
Show that the language
$$\{w \in \{a, b\}^* \mid \#_a(w) \text{ and } \#_b(w) \text{ are even}\}$$
is regular.
Easier is to show this for
$$\{w \in \{a, b\}^* \mid \#_a(w) \text{ is even}\}.$$