2. Regular languages, Finite Automata

Read: Chapter 2, pages 5–8 of the Reader Ruohonen; the slides of the course on the webpage.

Exercise 3 can be handed in with Nico Broeder or Jasper Derikx.

1. Let $M$ be the deterministic finite automaton (DFA) given by

$$
\langle Q, \Sigma, \delta, q_0, F \rangle
$$

with $\Sigma = \{a, b, c\}$, $Q = \{q_0, q_A, q_B, q_C\}$, $\delta$ given by the table

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$q_0$</th>
<th>$q_A$</th>
<th>$q_B$</th>
<th>$q_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>$q_A$</td>
<td>$q_0$</td>
<td>$q_C$</td>
<td>$q_B$</td>
</tr>
<tr>
<td>$b$</td>
<td>$q_B$</td>
<td>$q_C$</td>
<td>$q_0$</td>
<td>$q_A$</td>
</tr>
<tr>
<td>$c$</td>
<td>$q_C$</td>
<td>$q_B$</td>
<td>$q_A$</td>
<td>$q_0$</td>
</tr>
</tbody>
</table>

and $F = \{q_0\}$.

(a) Make a state transition diagram for $M$.

(b) Determine for the following words whether they belong to $L(M)$:

- $abba$
- $baab$
- $bac$
- $cac$

(c) Give a regular expression $e$ such that $L(e) = L(M)$.

2. Let a DFA $M$ be given by

(a) Describe the words accepted by $M$.

(b) Give a regular expression $e$ such that $L(e) = L(M)$.

3. $\Sigma = \{a, b\}$.

(a) Give a DFA that accepts $L_1 := \{w \in \Sigma^* \mid \text{\it w does not contain } ab\}$, and prove that your answer is correct.

(b) Give a DFA that accepts $L_2 := \{w \in \Sigma^* \mid \text{\it every } a \text{ in } w \text{ is directly followed by a } b\}$ and prove that your answer is correct.

(c) Give a DFA that accepts $L_3 := \{w \in \Sigma^* \mid \text{\it w contains } aa \text{ twice}\}$. (Be aware of $aaa$.)
