Exercises week 3

Languages and Automata

May 17, 2013

3. Regular languages, Finite Automata

Exercise 3.2 can be made and handed in; Exercise (*) is a bit harder. Let $\Sigma = \{a, b\}$.

3.1.
1. Construct a DFA M_1 such that

$$L(M) = L_1 = \{w \in \Sigma^* \mid \#_a(w) \text{ is divisible by } 3\}.$$

2. Construct an M_2 such that

$$L(M) = L_2 = \{w \in \Sigma^* \mid \#_b(w) \text{ is divisible by } 2\}.$$

3. Construct a NFA$_\lambda$ M_3 such that $L(M_3) = L_1 \cup L_2$.
4. Construct a DFA M_4 such that $L(M_4) = L_1 \cup L_2$.

3.2. Construct a regular expression e such that

$$L(e) = L = \{w \in \Sigma^* \mid \text{‘}abba\text{’ does not occur in } w\}.$$

We do this in several steps
1. First find an NFA M such that its language is $\overline{L}$.
2. Then construct a DFA M' accepting the same language.
3. Modify M' to obtain M'' accepting L.
4. Find e such that $L(e) = L(M'')$.

3.3. Assume that L over Σ is regular.
1. Show that $L_1 := \{w \in \Sigma^* \mid \exists v \in L(w \text{ is an initial segment of } v)\}$ is regular.
2. (*) Show that $L_2 := \{w \in \Sigma^* \mid \exists v \in L(w \text{ is contained in } v)\}$ is regular.

NB w is an initial segment of v if $v = wu$ for some u;
w is contained in v if all the symbols of w occur in v in that order, to be precise: $v = s_1 \ldots s_n$ and $w = s_{i_1} \ldots s_{i_m}$ for some sequence $1 \le i_1 < \ldots < i_m \le n$. ($s_j \in \Sigma$.)