Formal Languages, Grammars and Automata

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4. Non-regular languages and the pumping lemma

Exercise 4.3 can be made and handed in, (*) is an extra exercise. Let $\Sigma = \{a, b\}$.

4.1.
1. Construct a DFA that accepts L where $L = L(b^*aa^*bb^*(aa^*bb^*aa^*b)^*)$
2. Derive (just like in the pumping lemma) from your automaton a number k, for which you can prove:
   for every $w \in L$ with $|w| \geq k$, there are words u_1, v, u_2 such that
   - $w = u_1 v u_2$ and
   - $|v| \geq 1$,
   - $|u_1 v| \leq k$
   - $\forall n \in \mathbb{N}(u_1 v^n u_2 \in L)$.

   Prove this.

4.2.
1. Prove that the language L_1 is not regular, where

$$L_1 := \{a^n b^p \mid p > n\}$$

2. Prove that the language L_2 is not regular, where

$$L_2 := \{a^p b^n \mid p > n\}$$

   Try to give a proof without using the pumping lemma.

4.3. Now, $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, +, \times, (,)\}$ Prove that the language L_3 is not regular, where

$$L_3 := \{e \in \Sigma^* \mid e \text{ is a well-formed arithmetical expression}\}$$

NB. In a *well-formed arithmetical expression* the brackets should "match", so $3 \times (5 + (3 + 0))$ is well-formed and so is $((((4 + 5) \times 7)))$, but $5 + 9) + 3)$ and $(4 \times (3 \times 7)$ are not.

4.4. (∗) Do Exercise 5.4.2. from the Lecture Notes of Fiore (see the webpage of the course).