

Formal Languages, Grammars and Automata

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5. Context free Grammars

Exercise 5.5 can be made and handed in. Let $\Sigma = \{a, b\}$.

5.1. Consider the context-free grammars

$$G_1 \quad \begin{array}{l} S \rightarrow aABb \\ A \rightarrow aA \mid a \\ B \rightarrow bB \mid b \end{array} \quad G_2 \quad \begin{array}{l} S \rightarrow AAB \\ A \rightarrow AA \mid a \\ B \rightarrow BB \mid b \end{array}$$

- (i) Describe $L_1 = L(G_1)$ and $L_2 = L(G_2)$.
(ii) Show $L_1 = L_2$.

5.2. Construct a context free grammar G_1 such that

$$L(G_1) = \{a^p b^n \mid p > n\}$$

5.3. Construct a context free grammar G_2 such that

$$L(G_2) = \{w \in \Sigma^* \mid \#_a(w) \text{ is even}\}$$

5.4. Construct a context free grammar G_3 such that

$$L(G_3) = L((ab)^*(a \cup bb)^*).$$

5.5. $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, \times, +, (,), \}$. Construct a context free grammar G_4 such that

$$L(G_4) = \{w \in \Sigma^* \mid w \text{ is a well-formed arithmetical expressions}\}$$

NB. $2+3+4 \times 5$ and $((2+3)+4) \times 5$ and $((2+3)) + 4 \times 5$ are well-formed.
 $2 + (3 + 4 \times 5$ and $(2 + 3) + 4) \times 5$ and $)$ (are not.