## 6. Exercises Formal Grammars, Languages and Machines, Week 6, June 7, 2013

Exercise 6.2 can be made and handed in.
6.1. Consider $L_{1}=\{a b c\}$ and $L_{2}=L(M)$ with $M$ a NFA given by

| $\delta$ | $a$ | $b$ | $c$ |
| :---: | :---: | :---: | :---: |
| $q_{0}$ | $q_{0}, q_{1}$ |  |  |
| $q_{1}$ |  | $q_{1}, q_{2}$ |  |
| $q_{2}$ |  |  | $q_{0}, q_{2}$ |

in $M$ we have $F=\left\{q_{0}\right\}$.

Let

$$
\begin{aligned}
& L_{3}=\left\{a^{n} b^{n+2} \mid n \geq 0\right\} \\
& L_{4}=\left\{a^{n} b^{n} a^{m} b^{m} \mid n>0, m>0\right\}
\end{aligned}
$$

(i) Describe the language $L_{2}$ using a regular expression.
(ii) Give regular grammars for $L_{1}, L_{2}$.
(iii) Give a context-free grammar for $L_{3}$.
(iv) Give a context-free grammar for $L_{4}$.
6.2. Consider the regular grammars

$G_{1}$| S | $\rightarrow$ | $\mathrm{abA} \mid \mathrm{baB}$ |
| :--- | :--- | :--- |
| A | $\rightarrow$ | $\mathrm{aaA} \mid \mathrm{aS}$ |
| B | $\rightarrow$ | $\mathrm{bbB}\|\mathrm{bS}\| \lambda$ |


$G_{2}$| S | $\rightarrow$ | $\mathrm{aA}\|\mathrm{bS}\| \lambda$ |
| :--- | :--- | :--- |
| A | $\rightarrow$ | $\mathrm{bA} \mid \mathrm{aB}$ |
| B | $\rightarrow$ | $\mathrm{bB} \mid \mathrm{aS}$ |

(i) Give a $\mathrm{NFA}_{\lambda}$ that accepts $L_{1}=L\left(G_{1}\right)$.
(ii) Generate 4 different words using $G_{2}$.
(iii) Give a regular expression $e$ such that $L(e)=L\left(G_{2}\right)$.
6.3. (This is a challenge!) Give a context-sensitive grammar that generates

$$
\left\{a^{n} b^{n} c^{n} \mid n \geq 1\right\}
$$

This language is not context free.

