

# Exercises week 7

## Languages and Automata

June 14, 2013

### 7. Pushdown automata and context-free languages

Exercise 7.2 can be handed in.

7.1. Let  $M$  be the PDA with

$$\begin{array}{ll}
 Q = \{q_0, q_1, q_2\} & \delta(q_0, a, \lambda) = \{[q_0, A]\} \\
 \Sigma = \{a, b\} & \delta(q_0, \lambda, \lambda) = \{[q_1, \lambda]\} \\
 \Gamma = \{A\} & \delta(q_0, b, A) = \{[q_2, \lambda]\} \\
 F = \{q_1, q_2\} & \delta(q_1, \lambda, A) = \{[q_1, \lambda]\} \\
 & \delta(q_2, b, A) = \{[q_2, \lambda]\} \\
 & \delta(q_2, \lambda, A) = \{[q_2, \lambda]\}
 \end{array}$$

- (i) Draw a state diagram for  $M$ .
  - (ii) Show that  $aabb \in L(M)$  and  $aaab \in L(M)$ .
  - (iii) Verify whether  $aba$ ,  $abb$  and  $aab$  are in  $L(M)$ .
  - (iv) Describe the language accepted by  $M$ .
- 7.2. Construct a PDA that accepts the language  $L \subseteq \{a, b\}^*$ , where

$$L = \{w \mid \#_a(w) = \#_b(w)\}.$$

(So the number of  $a$ 's equals the number of  $b$ 's in  $w$ .)

7.3. Given is the grammar  $G_1$  over  $\Sigma = \{a, b, c\}$ :

$$\begin{array}{ll}
 S & \rightarrow aAD \\
 A & \rightarrow aAD \mid a \\
 D & \rightarrow bC \\
 C & \rightarrow c
 \end{array}$$

- (i) Construct a PDA accepting  $L(G_1)$ .
  - (ii) Describe the language  $L(G_1)$ .
- 7.4. (Harder) Show that the class of context-free languages is not closed under intersection.  
(Hint: use that the language  $L = \{a^n b^n c^n \mid n \geq 0\}$  is not context-free and devise two context-free languages  $L_1, L_2$  such that  $L = L_1 \cap L_2$ .)